

On the Peak Sets for Holomorphic Lipschitz Functions

JOAQUIM BRUNA

I. Introduction and notations. Let D denote the open unit disc in the complex plane and let T be its boundary. For $0 < \alpha \leq 1$, $\text{Lip } \alpha$ will denote the algebra of complex-valued functions f analytic on D , continuous on \bar{D} and satisfying a Lipschitz condition of order α on \bar{D} :

$$|f(z) - f(w)| \leq K|z - w|^\alpha, \quad z, w \in \bar{D}.$$

A closed set $E \subset T$ is said to be a *peak set* for $\text{Lip } \alpha$ if there exists $f \in \text{Lip } \alpha$ such that $f = 1$ on E and $|f| < 1$ on $\bar{D} \setminus E$ (and in this case we say that f peaks on E).

In [9], W. P. Novinger and D. M. Oberlin studied the peak sets for $\text{Lip } \alpha$. They showed that the peak sets for $\text{Lip } 1$ are just the finite sets. Moreover, if (ε_n) is the sequence of lengths of the complementary intervals of E in T , they proved that

$$(1) \quad |E| = 0, \quad \sum_n \varepsilon_n^{(1-\alpha)/(3-\alpha)} < +\infty,$$

is a sufficient condition and that

$$(2) \quad |E| = 0, \quad \sum_n \varepsilon_n^{1-\alpha} |\log \varepsilon_n|^{-\delta} < +\infty, \quad \delta > 1$$

is a necessary condition for E being a peak set for $\text{Lip } \alpha$. They also conjectured that

$$(3) \quad |E| = 0, \quad \sum_n \varepsilon_n^{1-\alpha} < +\infty$$

is a necessary and sufficient condition. If $\rho(z)$ denotes the Euclidean distance from z to E , we note that (3) means that $\rho^{-\alpha} \in L^1(T)$, whereas (1) means that $\rho^{-2/(3-\alpha)} \in L^1(T)$.

In this paper we continue this study and we offer some necessary conditions and some sufficient conditions which are very close to the conjecture. In particular, we show (Theorem 2.7) that $\rho^{-\alpha} \in L^{1+\delta}(T)$ for some $\delta > 0$ is a sufficient