

# *A Defect Relation for Meromorphic Maps*

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**Introduction.** Let  $f: \mathbf{C}^m \rightarrow \mathbf{P}_n$  be a meromorphic map. A well-known problem in value distribution theory is to find a good upper bound for  $\sum \delta_f(D_j)$ , where  $\delta_f$  is the Nevanlinna defect and  $\{D_j\}$  is any finite collection of hypersurfaces of degree  $d$ .

If  $\{D_j\}$  are hyperplanes it has been found that  $\sum \delta_f(D_j) \leq n + 1$ , assuming  $f$  is not contained in any hyperplane of  $\mathbf{P}_n$  and  $\{D_j\}$  are in general position (see Nevanlinna [8], Cartan [4], Ahlfors [1], Weyl [14], Stoll [11], Cowen-Griffiths [5], Vitter [13]).

For the general case we have the results of Carlson-Griffiths-King [3], [7] which imply  $\sum \delta_f(D_j) \leq (n + 1)/d$ , where  $m \geq n = \text{rank } f$  and where  $\{D_j\}$  have normal crossings.

Therefore as a natural generalization of the case  $d = 1$ , it has been conjectured (Griffiths [6], Shiffman [9]) that if  $\{D_j\}$  have normal crossings and if the image of  $f$  is not contained in any hypersurface of  $\mathbf{P}_n$ , then  $\sum \delta_f(D_j) \leq (n + 1)/d$ .

Until now, despite many attempts, the conjecture remains unresolved. However, some partial results have been obtained. For instance, Shiffman [10] established an upper bound of  $2n$  for the defects sum of a particular class of nonconstant meromorphic maps, requiring the  $\{D_j\}$  are such that no point of  $\mathbf{P}_n$  is contained in  $n + 1$  distinct  $D_j$ 's. Assuming  $\{D_j\}$  with normal crossings, Biancofiore [2] obtained an upper bound of  $(n + 1)/d$  for a class of algebraically nondegenerate meromorphic maps which essentially extends Shiffman's class.

In this paper, using the Veronese map, a defect relation is obtained for all meromorphic maps  $f: \mathbf{C}^m \rightarrow \mathbf{P}_n$  which are not contained in any hypersurfaces of degree  $d$ . We assume  $f$  is transcendental and  $\{D_j\}$  have normal crossings. Our result is

$$(1) \quad \sum \delta_f(D_j) \leq n + 1 + \mu(f)$$

where  $\mu(f)$  is an auxiliary defect which measures how far  $\sum \delta_f(D_j)$  differs from  $n + 1$ . We express  $\mu(f)$  explicitly, which is basically a limit of the quotient of differences of order functions. We observe that  $n + 1 + \mu(f)$  may well be less than  $\binom{n+1}{d}$ , the value expected from using the Veronese map. Moreover we