

Decomposition of Weakly Measurable Functions

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The principal result of this paper is that a bounded weakly measurable function f into a Banach space may be uniquely written as $f = g + h$ where g is strongly measurable and h is not strongly measurable in an extreme way, thus isolating the question of Pettis integrability of f to that of h , the extreme case.

In Section I we introduce the Stonian transform of a measurable function and sort out basic relationships. In Section II we specialize to Banach space valued functions and obtain the decomposition. Section III concludes with some brief remarks on lifting compactness and a generalized decomposition.

I. The Stonian transform of a measurable function. Let (Ω, Σ, μ) be a complete probability space and let S be the compact, Hausdorff Stone representation space of the complete Boolean algebra $A = \Sigma/\mu^{-1}(0)$. We denote the identification of $L^\infty(\Omega, \Sigma, \mu)$ with $C(S)$ by $f \rightarrow \hat{f}$, a norm isometry. The measure (again denoted by) μ lives on S as well as the unique representing Borel measure of the functional $\phi(\hat{f}) = \int_\Omega f d\mu$ on $C(S)$; thus, $\int_\Omega f d\mu = \int_S \hat{f} d\mu$. A set $N \subset S$ is nowhere dense iff $\mu^*(N) = 0$ and without further notice we write μ also for the extension of μ to the μ^* -measurable subsets of S .

The original motivation for this paper was an attempt to define a Stonian version of a weakly measurable function f on Ω into a Banach space X and from this to isolate Pettis integrability from Bochner integrability; see Section II. It was later realized that a more general transform was available and we introduce it now and then specialize to Banach spaces where the substantive result can be found.

Let T be a completely regular subspace of a compact Hausdorff space K . Let f be a function on Ω into T which is Baire measurable into K . We define the Stonian transform $\hat{f}: S \rightarrow K$ of f as follows:

Let $\hat{f}(s)$ be the unique point in K which represents the multiplicative linear functional $\Gamma_s(\xi) = (\widehat{\xi \circ f})(s)$ defined on $\xi \in C(K)$. Thus for all $\xi \in C(K)$, the equation

$$(\widehat{\xi \circ f})(s) = \xi(\hat{f}(s))$$

defines the function $\hat{f}: S \rightarrow K$ as a continuous function into K inducing the regular Borel measure $\hat{\nu} = \mu \hat{f}^{-1}$ on K . We call \hat{f} the *Stonian transform of f in K* .