

Existence Criteria for Capillary Free Surfaces without Gravity

ROBERT FINN

Introduction

This paper addresses the general question of how a bathtub should be designed for use in a spaceship. Specifically, one wishes to have a cylindrical container closed at one end and of finite height, which will hold, in the absence of gravity, a finite volume of liquid in mechanical equilibrium covering the base, and such that the (capillary) free surface meets the bounding cylinder walls in a prescribed angle γ . It was pointed out by Concus and Finn that a solution of the corresponding mathematical problem need not exist, even for containers with analytic bounding walls; experiments have verified the mathematical prediction.

Solution surfaces do exist in many circumstances; however, attempts to find explicit geometric criteria (Giusti, Weinberger, Chen, Finn) have met with only partial success. In Part A of the present paper, the problem is studied from a somewhat different point of view than has been adopted previously, leading to new criteria (e.g., Theorem 5.1) and a unified interpretation of past results. In Part B the criteria are applied to a specific situation, in which singular (paradoxical) behavior has been observed. The behavior is clarified in a way that leads (§13) to a qualitative description of the phenomenon in every situation in which it occurs.

As corollaries of the method, we obtain examples (§14, §11) to show that results obtained by Chen and by Chen and Huang for the case $\gamma = 0$, do not extend to any $\gamma > 0$.

A. General Considerations

1. On the earth's surface, the shape of the free surface in a capillary tube is determined by an interaction of surface and gravity forces. If a significant mass of fluid is present, the gravity forces will predominate; thus the effect of rising liquid is observed only in tubes of small diameter.

If gravity is removed, the effect becomes a consequence purely of surface forces, and geometrically similar free surfaces appear in geometrically similar containers. In a number of recent papers it has been shown that the behavior of the free surface can differ strikingly from what happens in a gravitational field. We intend in this