

The Cauchy Problem for Hyperbolic Operators with Non-smooth Characteristic Roots

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1. Introduction. In this paper, we shall study the Cauchy problem for some class of hyperbolic differential operators with non-smooth characteristic roots. A typical example of our class is as follows: Let $t \in \mathbf{R}$, $\ell \in \mathbf{Z}_+$, $x = (x', x'') \in \mathbf{R}^{p'+p''}$, $y \in \mathbf{R}^q$ and

$$(1.1) \quad P = D_t^2 + ((t^2 + |x'|^2)^\ell \Delta_x + \Delta_y + Q$$

where Q is a first order differential operator. In this example, since the characteristic roots of P are not smooth at $t = x' = \eta = 0$, it is impossible to prove a well-posedness of the Cauchy problem, using pseudo-differential operators with the symbols defined by characteristic roots. However, a difference of two characteristic roots is greater than

$$C_1((t^2 + |x'|^2)^\ell + |\eta|^2 r^{-2})^{1/2} r, \quad \text{where } r^2 = |\xi|^2 + |\eta|^2.$$

This fact suggests if we define a weighted symbol class associating to a degeneration of P , we can prove a well-posedness of the Cauchy problem for P , using a well-known energy inequality method like a strictly hyperbolic case, i.e., $\ell = 0$.

Using a technique of [6], which was used to construct a fundamental solution of the Cauchy problem for P in the case of $p' = q = 0$ (in this case, the characteristic roots are smooth), we may assume the characteristic roots of P are essentially smooth. By this fact and an energy inequality method used in [7], we shall prove the following.

Theorem for simple example. (See Theorem 5.2 and 6.2.) *The following two statements are equivalent:*

- i) *The Cauchy problem for P is well-posed in C^∞ and the solution has a finite propagation speed.*
- ii) *Let Q be $aD_t + bD_x + cD_y + d$, where a, b, c, d are smooth functions. Then $(D_t^j D_{x'}^{\alpha'}, b)(0, 0, x'', y) = 0$ if $j + |\alpha'| < \ell - 1$.*

Since a proof of the part from i) to ii) is in [3], in this paper we shall prove that statement ii) implies statement i).