

Sets with the Weak Radon-Nikodym Property in Dual Banach Spaces

LAWRENCE H. RIDDLE, ELIAS SAAB
& J. J. UHL, JR.

Dedicated to the memory of B. J. Pettis

Shortly after the appearance of Rosenthal's signal theorem on spaces containing ℓ_1 , a number of additional characterizations of such spaces appeared. They are collected below.

Theorem. *Each of the following statements characterizes Banach spaces X that contain no copy of ℓ_1 .*

(a) (Haydon). *Every x^{**} in X^{**} is universally (weak*-)-measurable and satisfies the barycentric formula on the unit ball of X^* equipped with the weak*-topology.*

(b) (Pelczyński). *Every bounded linear $T:L_1 \rightarrow X^*$ is a Dunford-Pettis operator.*

(c) (Musiał-Janika). *The dual X^* has the Radon-Nikodym property for the Pettis integral.*

(d) (Saab and Saab). *The restriction of each x^{**} in X^{**} to each non-empty weak*-compact subset of X^* has a point of weak*-continuity.*

The main goal of this paper is to localize these theorems by showing that statements (a)–(d) above localize to provide equivalent conditions for absolutely convex weak*-compact subsets of dual spaces.

By and large the work is based on an analogue of a factorization theorem of Stegall which shows, roughly, that weakly precompact sets are to sets with the weak Radon-Nikodym property as GSP sets are to sets with the Radon-Nikodym property. Included also is a new condition for universal Pettis integrability and special results for separable Banach spaces.

I. Weakly pre-compact sets. Throughout this paper X and Y are real Banach spaces with duals X^* and Y^* respectively. The closed unit ball in X will be denoted by B_X .

Definition. (a) A subset K of X is called weakly pre-compact if every bounded sequence in K has a weakly Cauchy subsequence.

(b) We say that a bounded sequence (x_n) in X is a copy of the usual ℓ_1 -basis