

The Kirchhoff Approximation for Maxwell's Equation

DAVID YINGST

Maxwell's equations for electro-magnetic fields in a vacuum can be decoupled yielding the following equations an electric field must satisfy

$$(0.1) \quad (\partial_t^2 - \Delta)\underline{E} = 0 \quad \operatorname{div} \underline{E} = 0.$$

(Here and throughout the underline notation denotes a vector field on a subset of \mathbf{R}^3 .) If a perfectly conducting obstacle, $K \subset \mathbf{R}^3$ compact, is imposed one then solves (0.1) in $\Omega = (\mathbf{R}^3 \setminus K) \times \mathbf{R}$ with the boundary condition

$$(0.1a) \quad \nu \times \underline{E}|_{\partial\Omega} = 0$$

where ν is the unit outer normal to K . For ∂K strictly convex we intend to study the solution of (0.1, 0.1a) given that \underline{E} is the plane field, $\underline{E} = \delta(t - x \cdot \omega)\underline{\theta}$ for $t \ll 0$ where $\omega \cdot \theta = 0$, $|\omega| = |\theta| = 1$.

More precisely let $\underline{E}_s = \delta(t - x \cdot \omega)\underline{\theta} - \underline{E}$ where $(\partial_t^2 - \Delta)\underline{E} = 0$ in Ω , $\nu \times \underline{E} = 0$ on $\partial\Omega$, and $\underline{E} = \delta(t - x \cdot \omega)\underline{\theta}$ for $t \ll 0$. Define \underline{e}_s to be the t -Fourier transform of \underline{E}_s

$$(0.2) \quad \underline{e}_s(x, \lambda) = \int_{-\infty}^{\infty} e^{-i\lambda t} \underline{E}_s(x, t) dt.$$

Because of the outgoing condition, $\underline{E}_s = 0$ for $t \ll 0$, \underline{e}_s satisfies the Sommerfeld radiation condition, $\underline{e}_s = O(|x|^{-1})$ and $\partial_r \underline{e}_s - i\lambda \underline{e}_s = o(|x|^{-1})$, $|x| \rightarrow \infty$, and since \underline{e}_s satisfies the reduced wave equation it follows from Green's formula that

$$(0.3) \quad \underline{e}_s(x, \lambda) = \int_{\partial K} [\underline{e}_s(y, \lambda) \partial_\nu G_\lambda(x - y) - \partial_\nu \underline{e}_s(y, \lambda) G_\lambda(x - y)] d\sigma(y),$$

where $G_\lambda(x) = |x|^{-1} e^{i\lambda|x|}$ is the Green's function for (outgoing) solutions to $(\Delta + \lambda^2)u = 0$ on \mathbf{R}^3 . Thus \underline{e}_s is determined once we know \underline{e}_s , and $\partial_\nu \underline{e}_s$ on ∂K . With $\underline{K}_D = e^{i\lambda x \cdot \omega} \underline{e}_s|_{\partial K}$ and $\underline{K}_N = e^{i\lambda x \cdot \omega} \partial_\nu \underline{e}_s|_{\partial K}$, the purpose of this paper is to show that \underline{K}_D and \underline{K}_N have complete asymptotic expansions in λ .

The plan of attack is essentially the same as that used in [5] to analyze the Kirchhoff approximation for the scalar wave equation with Dirichlet data. First a parametrix for (0.1) is obtained and appropriate Dirichlet and Neumann operators