

The Dirichlet Problem for the Biharmonic Equation in a C^1 Domain in the Plane

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Introduction

In this paper we solve the Dirichlet problem for the biharmonic equation $\Delta^2 u = 0$ on a bounded simply connected C^1 domain $\Omega \subset \mathbf{R}^2$ with boundary data in the space $L_1^p(\partial\Omega) \times L^p(\partial\Omega)$, $p > 1$. More precisely, we show that the solution can be represented as a multiple layer potential of the type introduced by Agmon [1].

The multiple layer potential for constant coefficient elliptic equations (of arbitrary order) in two variables with no lower order terms was introduced by Agmon [1]. Agmon used the multiple layer potential to solve the Dirichlet problem for elliptic equations of order $2m$ on domains contained in \mathbf{R}^2 with $C_{1+\beta}$ boundaries and $C_{m-1,\alpha}$ boundary data where $\alpha > 0$ and $\beta > 1/2$. In this paper we extend Agmon's result for the biharmonic operator to C^1 domains with L^p -type boundary data.

The Dirichlet problem for Laplace's equation in a C^1 domain in \mathbf{R}^n was solved by Fabes, Jodeit, and Rivière [4]. Their solution is in the form of a classical double layer potential in \mathbf{R}^n . While our proof follows the same general outline as [4] there are several significant differences. First of all, unlike Laplace's equation, Green's formula depends on the particular coordinate system. Thus in analyzing the multiple layer potential near the boundary one has to use a representation of it in the appropriate coordinate system. Secondly, in the case of Laplace's equation, crucial cancellation properties of the boundary kernel of the double layer potential needed to establish compactness of boundary integral operators were apparent from the simplicity of the kernels. In the biharmonic equation these cancellation properties are a consequence of special properties of the conjugate Neuman operators. These operators arise in the boundary integrals of Green's formula which in turn comes from what Agmon refers to as the "distinguished bilinear form." Thirdly, in Laplace's equation the boundary integral of the double layer potential maps $L^p(\partial\Omega)$ boundedly into itself, $1 < p < \infty$, and hence its adjoint can be realized as a bounded integral operator on $L^q(\partial\Omega)$, $1/p + 1/q = 1$. The adjoint is in fact realized as the normal derivative of the single layer potential at the boundary. In our case, the adjoint acts on the dual space of $L_1^p(\partial\Omega) \times L^p(\partial\Omega)$ which involves distributions. We are able to realize the adjoint operator