

Rotating Drops

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1. Introduction. The global energy of a liquid drop E which rotates around its own barycenter with constant angular velocity $\sqrt{2\Omega}$ under the combined action of surface and kinetic forces may be written as

$$(1.1) \quad \mathcal{F}_\Omega(E) = \int_{R^n} |D\varphi_E| - \Omega \int_E |y|^2 dydz.$$

As usual, we have indicated with φ_E the characteristic function of the set $E \subset R^n$ and with $\int_A |D\varphi_E|$, A an open set in R^n , the total variation of the vector valued measure $D\varphi_E$ (see [12], [27]):

$$(1.2) \quad \int_A |D\varphi_E| = \sup \left\{ \int_A \operatorname{div} \psi(x) dx, \psi \in [C_0^1(A)]^n, |\psi| \leq 1 \right\}.$$

We denote by $x = (y, z)$, with $y \in R^{n-1}$ and $z \in R$, an arbitrary point in R^n . H_k indicates the k -dimensional Hausdorff measure, while

$$\omega_n = H_n(\{x \in R^n : |x| \leq 1\}).$$

By φ_E^+ and φ_E^- we shall denote respectively the inner and outer traces of E on the boundary ∂A of the open set $A \subset R^n$ (see [28]). If the inner and outer traces are equal, the common value of φ_E^+ and φ_E^- will be denoted by φ_E .

We study the energy functional (1.1) in the class \mathcal{E} defined by

$$E \in \mathcal{E} \Leftrightarrow \begin{cases} H_n(E) = 1 \\ \int_E x_i dx = 0 \quad (i = 1, \dots, n) \end{cases}$$

that is, among the sets E with prescribed volume and barycenter.

The energy functional being unbounded from below in such a class, we must look for a *local minimum* for \mathcal{F}_Ω . To this aim, we put

$$C_R = \{(y, z) \in R^n : |y| < R, |z| < R\}$$

and we call $E \in \mathcal{E}$ a local minimum (a rotating drop) for \mathcal{F}_Ω if there exists $R > 0$ such that

- (i) $E \subset\subset C_R$