

# *On Area-Minimizing Hypersurfaces in Manifolds of Non-Negative Curvature*

MICHAEL T. ANDERSON

This paper is concerned with the question of existence and non-existence of complete area-minimizing hypersurfaces in complete, non-compact manifolds  $N^{n+1}$  of non-negative curvature. The fundamental result along these lines has become known as the “Bernstein Theorem”: a complete area-minimizing hypersurface in  $\mathbf{R}^{n+1}$  is a hyperplane, for  $n \leq 6$ . Counterexamples to the theorem for  $n \geq 7$  were found by Bombieri-DiGiorgi-Giusti [BDG] and later by Lawson [L1].

The analogous problem when  $N^{n+1}$  is compact is much simpler, as first noticed by Simons [S]: he showed in particular that there are no stable minimal hypersurfaces in (oriented) manifolds of positive Ricci curvature. For  $N^{n+1}$  non-compact, the existence and geometric implications of complete minimizing geodesics (the case “dual” to hypersurfaces) is well understood since the work of Toponogov [T] and Cheeger-Gromoll (CG1)]. Also, the fact that there are no compact, closed minimal varieties in manifolds of positive curvature is a simple consequence of the convexity theory developed in [CG1]. More recently, Fischer-Colbrie and Schoen [F-CS] have obtained very strong results in case  $N^{n+1}$  is a 3-manifold: for instance, they show that if  $N^3$  is of positive Ricci curvature, then  $N$  admits no stable complete minimal surfaces. (This turns out to be false in higher dimensions.)

As the main theorem of this paper, we prove the following non-existence result. For any given  $p \in N^{n+1}$ , let  $S_p(t)$  be the geodesic sphere of radius  $t$  centered at  $p$  and let

$$d_t = \text{diam } S_p(t) = \sup\{\text{dist}_N(x,y) : x,y \in S_p(t)\}$$

be the extrinsic diameter of  $S_p(t)$ . The quantity

$$\delta_N \equiv \overline{\lim}_{t \rightarrow \infty} \frac{d_t}{t}$$

is a measure of the asymptotic geometry of  $N^{n+1}$ ; it is not difficult to see that  $\delta_N$  is independent of the choice of  $p$ . We then have

**Theorem 1.** *Let  $N^{n+1}$  be a complete, simply connected non-compact manifold of non-negative sectional curvature. Then there is a constant  $\delta > 0$ , depending*