

On Isolated Singularities in Some Nonlinear Partial Differential Equations

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Introduction. The purpose of this paper is to study the local behavior of positive solutions of some nonlinear elliptic partial differential equations.

We will follow the established practice of using the same letter C (with or without subscript) to denote different constants depending on prescribed arguments.

P. L. Lions [10] has studied the local behavior of positive solutions of $\Delta u = -u^q$, $1 < q < n/(n - 2)$. He proved the following: *Let Ω denote the unit ball in \mathbf{R}^n , $n \geq 3$ and suppose $u \in C^2(\Omega - \{0\})$ and satisfies $\Delta u = -u^q$ in $\Omega - \{0\}$, $1 < q < n/(n - 2)$. Then, u has either a removable singularity or*

$$(1.1) \quad \frac{C_2}{\|x\|^{(n-2)}} \leq u(x) \leq \frac{C_1}{\|x\|^{(n-2)}} \quad \text{near the origin}$$

and

$$(1.2) \quad \Delta u = \alpha \delta_0 + u^q, \quad \alpha > 0 \text{ in } \mathcal{D}'(\Omega).$$

Moreover, he constructed solutions of (1.2).

Gidas and Spruck in [8], among other things, gave a complete description of the local behavior of positive solutions of

$$(1.3) \quad \Delta u + u^q = 0 \quad \text{with } \frac{n}{(n-2)} < q < \frac{n+2}{n-2}.$$

To be precise they proved the following: *Let $u > 0$, $u \in C^2(\Omega - \{0\})$ and suppose u satisfies (1.3) in $\Omega - \{0\}$. Then, u has either a removable singularity or*

$$(1.4) \quad \frac{C_2}{\|x\|^{2/(q-1)}} \leq u(x) \leq \frac{C_1}{\|x\|^{2/(q-1)}} \quad \text{near the origin.}$$

When $q = n/(n - 2)$, Gidas and Spruck [8] proved that $u(x) \leq C/\|x\|^{n-2}$. It was conjectured in [10, Remark 3] that the left-hand side of (1.4) should also occur (i.e., $u(x) \geq C/\|x\|^{n-2}$). Therefore, since necessarily $u \in L^{n/(n-2)}(\Omega)$ this estimate