

Homotopy Equivalences on Non-Irreducible 3-Manifolds

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§0. Introduction

This paper is concerned with classifying, up to homotopy, PL . Maps of non-irreducible 3-manifolds which induce inner automorphisms on the fundamental group. We obtain a complete classification of these maps when the manifold M is a connected sum of compact, P^2 -irreducible, sufficiently large 3-manifolds and at least one summand is not a (Möbius band) $\times S^1$ or a solid Klein bottle. In particular if f is a map of M which induces the identity automorphism on the fundamental group and is the identity on ∂M , then we show that f is homotopic (rel $\partial M \cup \{x_0\}$) to $r \circ d \circ k$ where r is a rotation about a sphere and d and k are homeomorphisms with support in collar neighborhoods of ∂M . Similar results have been obtained by H. Hendriks [3] and, for higher dimensions, D. McCullough [6], under the hypothesis that the manifolds have no boundary.

Denote by $E_{x_0}(M)$ the set of basepoint preserving, self-homotopy equivalences of M which are the identity on ∂M . Let $E^{\text{id}}(M)$ be those homotopy equivalences in $E_{x_0}(M)$ which induce the identity automorphism on the fundamental group and let $E^{\text{inn}}(M)$ be those homotopy equivalences in $E_{x_0}(M)$ which induce inner automorphisms on the fundamental group. We will prove

Theorem 3.3. *Let M be a connected sum of $t \geq 2$ compact, P^2 -irreducible, sufficiently large 3-manifolds. Suppose M contains an aspherical summand which is not a (Möbius band) $\times S^1$ or a solid Klein bottle. If M has s compressible torus boundary components, p incompressible torus boundary components, and q Klein bottle boundary components then*

$$\pi_0(E^{\text{id}}(M)) \cong \left(\bigoplus_{t-1} \mathbf{Z}_2 \right) \oplus \left(\bigoplus_s \mathbf{Z} \right) \oplus \left(\bigoplus_p (\mathbf{Z} \oplus \mathbf{Z}) \right) \oplus \left(\bigoplus_q \mathbf{Z} \right).$$

Theorem 3.6. *Let M be a connected sum of $t \geq 2$ compact, P^2 -irreducible, sufficiently large 3-manifolds. Suppose M contains an aspherical summand which is not a (Möbius band) $\times S^1$ or a solid Klein bottle. If M has s compressible torus boundary components, p incompressible torus boundary components, and q Klein bottle boundary components then*