

Stationary Solutions and Asymptotic Behavior of a Quasilinear Degenerate Parabolic Equation

M. SCHATZMAN

1. Introduction

This paper is concerned with a study of the equation

$$(1.1) \quad \frac{\partial u}{\partial t} - \Delta\phi(u) = f \quad u, x \in \mathbf{R}^N, \quad t \geq 0,$$

where ϕ is increasing, u is nonnegative, and f depends only on the spatial variable x ; we shall require f to take positive values at some point of \mathbf{R}^N , and to be negative for large values of $|x|$.

This kind of equation was introduced by Gurney and Nisbet [15], using probabilistic considerations to model the evolution of a biological population whose density is u , in an environment which is hostile or favorable according to the sign of f . In [15], Gurney and Nisbet took $\phi(u) = u^2$.

In more recent papers, Gurtin and MacCamy [16,17] introduced a more general equation

$$(1.2) \quad \frac{\partial u}{\partial t} - \Delta\phi(u) = f(x, u)$$

which they devised through concepts from continuum mechanics.

The common feature of these two papers is the assumption that the nonlinear diffusion term $\Delta\phi(u)$ describes a diffusion that avoids crowding. The right-hand term, which describes the effect of the environment on the population is positive when the environment is favorable, and negative when the environment is hostile.

Gurney and Nisbet announced in [15] that if f is positive on a bounded region, and negative elsewhere, there exists a stationary solution which is nonnegative, not identically zero, and compactly supported. Unfortunately, the proof of this result is only sketched.

In [19], Namba exhibits a stationary solution of (1.1), with $\phi(u) = u^2$, and $f(x) = A - Bx^2$, $x \in \mathbf{R}$. This solution is given by