

Collectively Measurable Sets and Abstract Wiener Spaces

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Suppose that n is the Gauss measure on a real Hilbert space H . Then n is not σ -additive on the algebra of cylinder sets in H when it is infinite-dimensional, but it may happen that if H is completed in some way, then the cylindrical probability n extends to a Borel measure on the completed space.

The classical example is the space H of absolutely continuous functions f for which f' is square-integrable on the unit interval. The inner product of H is given by

$$(f, g) = \int_0^1 f'(t)g'(t) dt, \quad f, g \in H.$$

If H is completed with respect to the sup-norm, then the completed space is isomorphic to the space of continuous functions vanishing at zero, and the measure n extends to the Wiener measure.

The notion of an abstract Wiener space was formulated by L. Gross [3] to formalize this method. Extensions have been considered by A. Badrikian and S. Chevet [1], and R. M. Dudley, J. Feldman and L. Le Cam [2]. In the example above, the inclusion i of H in the completed space B is not “radonifying” in the sense of Schwartz [9], but it maps the *particular* cylindrical measure n into a regular measure. For a *given* cylindrical measure μ on H , the idea is to construct the completed space B and the inclusion $i: H \rightarrow B$ such that $\mu \circ i^{-1}$ is regular on B .

The completion with respect to a “measurable seminorm” is central to the construction of abstract Wiener spaces in the locally convex space case. Thus, if μ is a cylindrical probability on the locally convex space E , then μ extends to a Borel measure on the Banach space obtained via the μ -measurable seminorm p . However, no such extension may exist if the completion is taken with respect to an uncountable collection of μ -measurable seminorms defining a locally convex topology on E [2].

This problem may be circumvented by imposing restrictions on the new topology of E as Badrikian and Chevet [1] have done. Alternatively, we can work with the bounded sets of a bornology instead of seminorms and neighbourhoods of zero of a topology. Regularity inducing extension theorems for cylindrical