

Weak and Strong Solutions in Two Dimensions

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Introduction. We shall consider systems

$$(1) \quad Lu = (A_1 \partial_1 + A_2 \partial_2 + \tilde{\kappa})u = f$$

of first order linear partial differential equations on a domain $\Omega \subset \mathbf{R}^2$ with a piecewise smooth boundary $\Gamma = \bigcup \Gamma_j$, where the Γ_j 's are the smooth pieces of Γ . For simplicity, and without loss as far as our results are concerned, we assume that $\Gamma = \Gamma_1 \cup \Gamma_2$, and that the corner is at the origin. Let us assume that $A_j \in \mathcal{C}^2(\bar{\Omega})$ and is symmetric, $j = 1, 2$, that $\tilde{\kappa} \in \mathcal{C}(\bar{\Omega})$, and that A_1, A_2 , and $\tilde{\kappa}$ are constant outside of some sphere. Homogeneous boundary conditions will be assigned of the form

$$(2) \quad u \in N(x), \quad x \in \Gamma, \quad N(x) \text{ a subspace,}$$

or equivalently, $P(x)u(x) = 0$, where P is an idempotent matrix function which is in \mathcal{C}^2 on Γ_1 and on Γ_2 , but may jump across the corner. L acts on functions $u \in C_0^2(\bar{\Omega})$; we denote the restriction of L to functions satisfying (2) by L^0 or, to specify the boundary condition, by L_N^0 .

Denote by ν the unit exterior normal on Γ , and set $A_\nu = \sum A_j \nu_j$. We shall assume that near the corner, Γ is non-characteristic (A_ν^{-1} is uniformly bounded), and that functions $u \in N$ satisfy uniformly in x either

$$(3) \quad u \cdot A_\nu u \geq c_0 u \cdot u, \quad c_0 > 0, \quad \text{or}$$

$$(4) \quad u \cdot A_\nu u \geq 0.$$

Finally, N must be a maximal subspace with respect to (3) or to (4).

The maximal (weak) extension L_w^0 of L^0 is defined by setting $L_w^0 u = f$ provided

$$(5) \quad (f, \nu)_\Omega - (u, L^* \nu)_\Omega = 0$$

for all $\nu \in C_0^2(\bar{\Omega})$ which satisfy the homogeneous adjoint boundary condition $\nu \in N^* \equiv (A_\nu N)^\perp$; here L^* is the formal adjoint of L . The minimal (strong) extension L_s^0 of L^0 is the closure of L^0 with respect to the graph norm in L^2 . We shall study when $L_w^0 = L_s^0$, at first restricting attention to the two-by-two elliptic case and to the three-by-three elliptic-hyperbolic case. In the two-by-two case,