

Jets with Two Fluids I: One Free Boundary

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Introduction. In this work we consider the irrotational flow of two ideal fluids. In terms of the stream function u , when suitably normalized, the problem for plane flows is to solve

- (0.1) $\Delta u = 0$ in each fluid,
- (0.2) $u = 0$ on the free boundary Γ separating the two fluids,
- (0.3) $|\nabla u|^2$ has constant jump λ across Γ ,
- (0.4) $u = \text{const.}, |\nabla u| = \text{const.} = \lambda_1$ on the free boundary Γ_0
between fluid and air,

subject to boundary conditions, for example $u = \text{const.}$ on each fixed boundary. For 3-dimensional axially symmetric jets with the x -axis as the axis of symmetry, one replaces in (0.1) Δ by $\Delta - \partial/\partial y$ and in (0.3), (0.4) $|\nabla u|$ by $|\nabla u|/y$. Both λ and λ_1 are given constants of integration in Bernoulli's law. For jet problems one considers these constants as parameters which have to be chosen such that the free boundaries start at given points on the fixed boundary, the endpoints of the nozzle.

Our approach to solving the jet problem is based on a variational principle which yields the above conditions (the condition in (0.3) is satisfied in a weak sense). In the case of one fluid the jet problem was studied by the authors in [2], [3], [4], based on the general treatment of such variational principles in [1].

In [5] we have studied some general properties of variational problems with two phases (analogous to those established in [1]). These properties are mostly of local nature and will be used in order to study the solutions as they depend on λ and λ_1 . Of course we shall also use here some of the tools developed in [2], [3], [4].

In the present paper we consider only the case of one free boundary, namely, the boundary Γ between the two fluids with jump constant λ . This models a flow of two jets in an infinitely long pipe. In a subsequent paper we shall treat the case of two jets with two free boundaries.