

# *Subnormality of the Cesàro Operator and a Semigroup of Composition Operators*

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*Dedicated to the memory of James P. Williams, who introduced  
me to operator theory.*

**1. Introduction.** In their 1965 paper [1] Brown, Halmos, and Shields instituted the study of the classical Cesàro operators from the modern operator theoretic point of view, and proved, among other things, that the discrete Cesàro operator (defined below) is hyponormal. In 1971, Kriete and Trutt [11] proved that the discrete Cesàro operator is subnormal, and in 1972, Deddens [3] used this and a description of commutants due to Shields and Wallen [12] to show that certain composition operators on  $H^2$  of the unit disk are co-subnormal. In this paper, we will reverse this procedure: We will study certain composition operators, prove directly that they are subnormal, and this will enable us to prove that the Cesàro operator is subnormal. In addition, we are led to a Volterra integral operator on  $L^2(0, \infty)$  that is unitarily equivalent to the discrete Cesàro operator.

It is not one of the goals of this paper to avoid computation. However, one can construct a relatively noncomputational proof of the subnormality of the Cesàro operator from this material as follows. In Section 2, a strongly continuous semigroup of composition operators is constructed and in Section 3 each operator in the semigroup is shown to be subnormal. A theorem of Ito [9, page 10] shows that there is a normal semigroup that extends our semigroup, and since the infinitesimal generator of our semigroup is the negative of the inverse of the Cesàro operator, we may conclude that the Cesàro operator is subnormal. The difficulty with this approach is that one ends up not having computed the measure associated with the Cesàro operator.

The approach taken here is motivated by the work on weighted translation semigroups of Frankfurt [7] (which more explicitly describes the relevant measures than do Embry and Lambert [4], [5]). It turns out that the computations involving measures flow more naturally than those in Kriete and Trutt [11]; the theorem they use to show the existence of a measure [11, Section 3] is replaced by an operator theoretic one [7, page 1238]. Some of these computations parallel the work of Klopfenstein [10].

As we will be working in several different contexts, we need to establish some notation. The discrete Cesàro operator,  $A$ , is the operator on  $\ell^2$  given by