

On Distal Flows of Finite Codimension

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§0 Introduction. This paper was motivated by DeRiggi and Markley's paper on codimension one flows—free minimal actions of R^n on compact $(n + 1)$ -dimensional manifolds without boundary. In [DM] they show that if a codimension one flow is distal, then it is almost periodic. We prove several generalizations of this by using a refinement due to Bronstein, and also to Rees, of Furstenberg's structure theorem for distal minimal flows—a distal minimal flow may be obtained as a tower of flows X_i and homomorphisms $\phi_i^{i+1}: X_{i+1} \rightarrow X_i$, $i \leq \theta$ where X_0 is the singleton flow, $X_\theta = X$, and the ϕ_i^{i+1} are almost periodic homomorphisms (in the refinement, quotient Lie group homomorphisms). By this approach the codimension p case for minimal distal flows is easily handled—basically there is a tower with $\theta \leq p$.

We would like to thank Markley for telling us about the refinements of Bronstein and of Rees.

§1 Distal flows. We first establish our notation and summarize some basic known results. We will often use X to denote the flow (X, T) as well as the phase space. For convenience we will assume all topological spaces, including X and T , are separable, locally compact, and metric. These conditions are needed for most of the arguments we use. A homomorphism $\phi: X \rightarrow Y$ is assumed to be surjective.

$$\text{Let } R(\phi) = \{(x, x') : \phi(x) = \phi(x')\}.$$

The relative (or ϕ) proximal relation

$$P(\phi) = \{(x, x') \in R(\phi) : \lim xt_n = \lim x't_n \text{ for some net } t_n \text{ in } T\}.$$

A point x is said to be a ϕ -distal point if $\{x' : (x, x') \in P(\phi)\} = \{x\}$. We say ϕ is distal if every point in X is ϕ -distal. We call X distal if the map $\phi_0: X \rightarrow \{0\}$ is distal (here $\phi_0(x) \equiv 0$). The smallest closed invariant equivalence relation containing $P(\phi)$ is called the relative distal structure relation and is denoted by $S_d(\phi)$. The induced homomorphism of $X/S_d(\phi)$ onto Y is distal. The relative regional proximal relation

$$Q(\phi) = \{(x, x') : \text{there exist nets } x_n, x'_n \text{ in } X \text{ and } t_n \text{ in } T$$

$$\text{such that } x_n \rightarrow x, x'_n \rightarrow x', x_n t_n \rightarrow x, x'_n t_n \rightarrow x \text{ and } (x_n, x'_n) \in R(\phi)\}.$$