

Jets with Two Fluids II: Two Free Boundaries

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Introduction. This paper is a continuation of [5]; in that paper we studied symmetric plane flows and axially symmetric flows of two jets in a pipe, with one free boundary Γ_2 separating the fluids. In the present paper we shall study the symmetric plane flows of two jets where the outer jet is surrounded by air so that the flow region has two free boundaries, Γ_1 and Γ_2 , where Γ_1 is the boundary of the outer fluid.

This problem involves two unknown parameters λ_1, λ_2 ; λ_1 represents the velocity of the fluid at the outer boundary Γ_1 and $\lambda_2^2 - \lambda_1^2$ is the jump of the squares of the velocities of the two fluids across Γ_2 . The crucial task here is to show that these parameters can be determined so that the free boundaries Γ_1 and Γ_2 start precisely at the endpoints of the nozzles; this is the “continuous fit” condition. As in [5], continuous fit implies smooth fit, that is, the free boundaries start in the same direction as the tangents to the nozzles at their common endpoints, and the velocities are bounded near these points.

Physical problems involving several jets are described in [6], [7]. In [8] an existence and uniqueness theorem is established for two jets with two free boundaries in the case of plane symmetric flows. However, the nozzles are linear rays and the inner fluid is “very thin” and “very fast moving” so that a solution can be constructed by a convergent power series.

§1. The jet problem; the main result. We consider two nozzles N_1 and N_2 satisfying:

- (1.1) N_i lies in $\{x \leq 0, y > 0\}$ and it is an x -graph, and N_1 lies above N_2 ; more precisely, every line $\{x = c\}$ with $c \leq 0$ intersects N_i in either one point or one (nonempty) closed segment, and $y_1 > y_2$ if $(c, y_i) \in N_i$;
 $\{x = 0\}$ intersects N_i at one point, $(0, b_i)$;
 N_i is continuous and piecewise C^2 curve.

Set

$$A = (0, b_1), \quad B = (0, b_2), \quad b = \frac{b_1 + b_2}{2}.$$