

# *Blaschke Conditions for Holomorphic Mappings*

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**1. Introduction.** Suppose  $f: D \rightarrow \mathbf{C}$  is a bounded holomorphic function defined on the unit disk. Let  $\{a_\nu\}_{\nu=1, \infty}$  denote the zeros of  $f$ . It then follows that

$$(1.1) \quad \sum_{\nu=1}^{\infty} (1 - |a_\nu|) < +\infty.$$

There is thus a restriction on how fast the set of zeros of a bounded analytic function in  $D$  can grow as one approaches the boundary of  $D$ . We give a proof of (1.1) which will provide some motivation for the results in Section 4.

For  $f: D \rightarrow \mathbf{C}$  let  $n(t, 0) = \text{card}(f^{-1}(0) \cap D(t))$  where  $D(t) = \{z: |z| < t\}$ . Assuming  $f(0) \neq 0$  for simplicity, the integrated counting function of  $f$  is defined by

$$(1.2) \quad N(r, 0) = \int_0^r \frac{n(t, 0)}{t} dt.$$

The First Main Theorem of Nevanlinna then states

$$(1.3) \quad N(r, 0) + m(r, 0) = T(r)$$

where  $m(r, 0) \geq 0$  and  $T(r)$ , the order function, is bounded for a bounded function. It follows that for a bounded function  $N(r, 0)$  remains bounded as  $r \rightarrow 1$ . Using this fact we obtain (1.1) as follows.

$$(1.4) \quad \sum_{\nu=1}^{\infty} (1 - |a_\nu|) = \int_0^1 (1 - t) dn(t, 0) \leq \sup_{r < 1} \left[ (1 - r)n(r, 0) + \int_0^r n(t, 0) dt \right].$$

Therefore

$$(1.5) \quad \begin{aligned} \sum_{\nu=1}^{\infty} (1 - |a_\nu|) &\leq \sup_{r < 1} \left[ (1 - r)n(r, 0) + \int_0^1 \frac{n(t, 0)}{t} dt \right] \\ &\leq \sup_{r < 1} [(1 - r)n(r, 0) + \mathcal{O}(1)]. \end{aligned}$$