

Weighted Norm Inequalities for Classes of Operators

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1. Given indices p and q , $1 \leq p \leq q \leq \infty$ and a sublinear operator T defined on suitable function spaces and satisfying some weak type boundedness conditions, then sufficient conditions on non-negative pairs of functions u, v are given, such that

$$(1.1) \quad \left(\int_{\mathbf{R}^n} |u(x)Tf(x)|^q dx \right)^{1/q} \leq C \left(\int_{\mathbf{R}^n} |v(x)f(x)|^p dx \right)^{1/p}$$

holds. Here C is a constant independent of f and the inequality is interpreted in the sense that if the right side is finite, so is the left side and the inequality holds.

One of the main results (Theorem 2.4, Corollary 2.5) is a weighted Marcinkiewicz interpolation theorem with weights satisfying certain easily verifiable conditions. The result may also be viewed as an extrapolation theorem in the sense that if T is simultaneously of weak type (p_i, q_i) , $i = 0, 1$, then T satisfies a weighted strong type inequality of the form (1.1) for $1 \leq p \leq q \leq \infty$, where $1/p$ and $1/q$ need not be the usual convex combinations of $1/p_i$ and $1/q_i$, $i = 0, 1$.

Although weighted norm inequalities for many classical operators have been obtained [10], [13] by studying the kernels of the operators, we obtain in this note weighted estimates with mixed norms solely from the initial weak boundedness of the operator and weight conditions. Estimates involving weights by interpolation were already obtained by Stein [15], Rooney [11] and recently, Sagher [12] obtained estimates with general weights, similar to those of Theorem 2.4 (2.5) with $p = q$. Instead of his more abstract formulation via the Peetre K -functional we utilize for simplicity the Calderón estimate for operators of this type. In this way we see that for those operators which are of (weak) type $(1, \infty)$ and $(2, 2)$, for example the Fourier transform, the conditions on the weights can be considerably simplified. For p and q not both equal to 2, Muckenhoupt [8] as well as Jurkat and Sampson [6] gave different but equivalent [9] conditions on the pair of weights (u, v) for which the Fourier operator satisfies a weighted norm inequality of the form (1.1). Under additional monotonicity assumptions on the weights it is also shown in [8] that the conditions on (u, v) are also necessary. The necessary part of Theorem 3.1 also requires monotonicity of the weights so that in this case our conditions are equivalent to those in [6] and [8]. The suffi-