

Spectral Singularities of Dissipative Schrödinger Operators with Rapidly Decreasing Potential

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Dedicated to V. P. Havin on his fiftieth anniversary

Introduction. A convenient tool for investigation of spectral properties of dissipative operators is the Sz-Nagy and Foias model. According to this approach every simple dissipative operator can be described in terms of a characteristic function, i.e. a special operator-valued function S which is holomorphic and contractive in the open upper half-plane $\mathbf{C}_+ = \{\lambda : \text{Im } \lambda > 0\}$. From the point of view of complex function theory an interesting question is: what conditions imply that a given holomorphic and contractive function S in \mathbf{C}_+ is the characteristic function of a dissipative operator from a given class of dissipative operators?

This question is also important for the theory of spectral expansions of dissipative operators [13]. Since the spectral expansion of a dissipative operator is non-orthogonal, it is not always easy, compared with the self-adjoint case, to understand in what sense the expansion converges. A key role is played by the zero-set

$$S^{-1}(0) = \{k \in \mathbf{C} : \text{Im } k \geq 0, S(k) = 0\}$$

which is called a set of spectral singularities [13].

This paper deals with the following class of Schrödinger operators. Let V be a bounded and continuous real-valued function on $\mathbf{R}_+ = [0, +\infty)$ and let $h \in \mathbf{C}_+$. Then ℓ_h is a dissipative operator in $L^2(\mathbf{R}_+)$ defined by the differential expression

$$\ell y = -y'' + V \cdot y$$

and the boundary condition

$$y'(0) - hy(0) = 0.$$

For $\alpha > 0$ let \mathfrak{B}_α denote the class of all continuous real-valued functions on \mathbf{R}_+ satisfying

$$|V(r)| \leq C_v \cdot \exp\{-c_v \cdot r^{\alpha/(1+\alpha)}\}, \quad r \in \mathbf{R}_+,$$

where $C_v, c_v > 0$. Let G_α be the Gevrey class of order α , i.e. the class of all