

The Equivariant J-homomorphism for Arbitrary S^1 -actions

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Introduction. Let $G = S^1$, the circle group, act on \mathbf{C}^m in a unitary fashion, and denote by $\text{Map}_G(S^{2m-1}, S^{2m-1})$ the space of G -equivariant maps with the compact-open topology. When the action is free, W. Browder proved in [5] that the J -homomorphism

$$\pi_* U_m \rightarrow \pi_* \text{Map}_G(S^{2m-1}, S^{2m-1})$$

is injective modulo finite groups. This result was developed further in [1]. The object of this note is to investigate the analogous problem for arbitrary unitary actions. In this case the subspace of linear G -maps is a product $U_{m_1} \times \dots \times U_{m_k}$ of unitary groups corresponding to the various characters of the representation $G = S^1 \rightarrow U_m$. One can easily see that one cannot hope for a result analogous to Browder's without further restriction on the space of maps. It turns out that the appropriate space is the subspace

$$\text{Map}_G^0(S^{2m-1}, S^{2m-1}) \subset \text{Map}_G(S^{2m-1}, S^{2m-1})$$

of those maps whose restriction to the fixed-point set $(S^{2m-1})^G$ is the identity. It will be shown that if

$$\alpha = \sum \alpha_i \nu_i,$$

where $\alpha_i \in \mathbf{Z}$, $\nu_i \in \pi_{2r-1} U_{m_i}$ and $m_i \geq r$ is in the kernel of

$$\pi_{2r-1}(U_{m_1} \times \dots \times U_{m_k}) \rightarrow \pi_{2r-1} \text{Map}_G^0(S^{2m-1}, S^{2m-1}), \quad m_i \geq r$$

then a certain weighted linear combination in the coefficients α_i must vanish. In particular this implies that the homomorphisms

$$\pi_* U_{m_i} \rightarrow \pi_* \text{Map}_G^0(S^{2m-1}, S^{2m-1}), \quad i = 1, \dots, k,$$

are injective modulo finite groups.

The method of proof is similar to Browder's [5]. It depends on the defining relation of a G -invariant vector bundle.

The questions studied here arose in the course of H. Berestycki's and J. M. Lasry's study of the modified Seifert conjecture for conservative systems [2], [3]