Truncations of BMO Functions

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§1. Introduction and preliminaries. In dealing with function spaces containing unbounded functions one often proves theorems for the bounded functions in the space, then passes to the general case by some sort of limiting argument. One way this is done is by truncating the function, that is by chopping it off above a certain height, applying the result for bounded functions, then letting the truncation height go to infinity. In this paper we will study this process on a particular function space, the space BMO of functions of bounded mean oscillation. It turns out that on this space truncations behave especially nicely, and one can use them to show that as far as certain questions are concerned BMO functions look the same as bounded functions.

The following definitions and notations will be used in the sequel.

Definitions. If f is a complex-valued function on some space and $\rho > 0$,

$$T_\rho f(x) = \begin{cases} f(x) & \text{if } |f(x)| \le \rho \\ \rho \dfrac{f(x)}{|f(x)|} & \text{if } |f(x)| > \rho. \end{cases}$$

$$\mathbf{R}^{d+1}_+ = \{z = (x,y): x \in \mathbf{R}^d, y > 0\}.$$

For $z = (x,y) \in \mathbf{R}^{d+1}_+$ we denote by P_z the Poisson kernel for z:

$$P_z(t) = c_d \frac{y}{(|x - t|^2 + y^2)^{(d+1)/2}}$$

where c_d is chosen so that $\int_{\mathbf{R}^d} P_z(t)dt = 1$. We denote by $d\mu_z$ the probability measure $d\mu_z(t) = P_z(t)dt$. For a function f on $\mathbf{R}^d$ we denote by $f(z)$ the harmonic extension of f, $f(z) = \int_{\mathbf{R}^d} f\, d\mu_z$. The space H^1 is the usual real-variable Hardy class studied in [10]. For a set $E \subset \mathbf{R}^d$, $|E|$ denotes the Lebesgue measure of E. The space BMO is the set of functions f for which

$$\|f\|_* = \sup_{|Q|} \frac{1}{|Q|} \int_Q |f - f_Q| < \infty$$

where Q denotes a cube in $\mathbf{R}^d$ with edges parallel to the coordinate axes and $f_Q = 1/|Q| \int_Q f$. It is a well-known consequence of the John-Nirenberg Theorem

Indiana University Mathematics Journal ©, Vol. 33, No. 5 (1984)