

# *The Local Imbedding Problem for Conformal Connections*

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**Introduction.** Let  $(M, g)$  be a Riemannian manifold imbedded in the  $N$ -dimensional Euclidean space  $\mathbf{R}^N$ . It is well known that the Euclidean connection on  $\mathbf{R}^N$  canonically induces an affine connection on  $M$ . This connection agrees with the Levi-Civita connection of  $g$  if the imbedding is isometric. The Cartan-Janet theorem [3], [6] asserts that the Levi-Civita connection of any  $n$ -dimensional real analytic Riemannian manifold  $M$  may be locally induced from the standard flat Riemannian connection by a (local) real analytic imbedding of  $M$  into  $\mathbf{R}^N$  for  $N = (1/2)n(n + 1)$ .

The projective analogue of this result was proven by S. S. Chern [4]. Let  $M$  be an  $n$ -dimensional real analytic manifold with a real analytic, torsion free projective connection whose curvature tensor satisfies an analogue of the first Bianchi identity. Then Chern's result asserts that  $M$  may be locally mapped by a real analytic imbedding into the  $N$ -dimensional projective space  $P^N$ ,

$$N = \frac{1}{2}n(n + 1) + \left\lceil \frac{n}{2} \right\rceil,$$

in such a way that the given projective connection agrees with that induced by a suitable choice of normal framing from the standard flat projective connection on  $P^N$ .

The purpose of this paper is to prove the following analogue of these results in conformal geometry.

**Theorem.** *Let  $M$  be an  $n$ -dimensional real analytic manifold endowed with a real analytic torsion free conformal connection  $\omega$  whose curvature tensor satisfies the Bianchi identities (2.8). Then  $M$  may be locally mapped by a real analytic imbedding into the  $N$ -dimensional Möbius space  $S^N$  in such a way that  $\omega$  coincides with the conformal connection canonically induced on  $M$  by the standard flat conformal connection on  $S^N$ .*

In this theorem we require

$$N = \frac{1}{2}n(n + 1) + \left\lceil \frac{n}{2} \right\rceil,$$