

# *Scattering Theory for the Wave Equation with a Short Range Perturbation. II*

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**6. Introduction.** A general treatment of scattering theory for the wave equation with a short range potential in spaces of dimension  $n \geq 3$  was presented in Part I of this work [8]. In Part II we give a parallel development for dimensions one and two.

We again treat the wave equation:

$$(6.1) \quad u_{tt} = Lu = \partial_i a_{ij} \partial_j u - qu,$$

where the matrix  $(a_{ij}(x))$  is  $C^{(1)}$  and everywhere positive,  $q$  is locally  $L_2$ , and for  $r = |x| > R$

$$(6.2) \quad \begin{aligned} |a_{ij} - \delta_{ij}| &\leq C/r^\alpha \geq |\partial a_{ij}| && \text{for some } \alpha > 1, \\ |q| &\leq C/r^\beta && \text{for some } \beta > 2. \end{aligned}$$

We also require the unique continuation property for the eigenfunctions of  $L$ .

We take for the unperturbed equation

$$(6.3) \quad u_{tt} = L_0 u = \Delta u,$$

with initial data  $u(x,0) = f_1(x)$  and  $u_t(x,0) = f_2(x)$ . Denoting the data pair  $\{f_1, f_2\}$  by  $f$ , the energy form for (6.3) is

$$(6.4) \quad E_0(f) = \int (|\nabla f_1|^2 + |f_2|^2) dx.$$

The essential difference between  $n \geq 3$  and  $n < 3$  arises from the fact that when  $n = 1$  or  $2$  the completion with respect to  $E_0$  of  $C_0^\infty(R^n)$  data contains data of the form  $\{c, 0\}$ . Denoting the solution operators for the perturbed and unperturbed wave equations by  $U$  and  $U_0$ , respectively, it is easy to see that

$$(6.5) \quad U_0(t)\{c, 0\} = \{c, 0\}$$

and hence, for a nontrivial perturbation, the limit as  $t$  becomes infinite of  $U(-t)U_0(t)\{c, 0\}$  does not exist.

Since the wave operators

$$(6.6) \quad W_\pm = \text{st} \lim_{t \rightarrow \pm\infty} U(-t)U_0(t)$$