

Removable Singularities of Harmonic Maps

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1. Introduction. A subset A of a Riemannian manifold M is *polar* if any L_1^2 -function defined on a domain $U \subset M$ and harmonic on $U \setminus U \cap A$ is harmonic on U . That notion can be characterized in terms of capacity (see §2), and is thereby seen to be independent of the Riemannian metric on M .

With explanations and refinements to follow, we state now the

Theorem. *Let A be a polar set of M . Let $f \in L_{1,\text{loc}}^2(M, N)$ be a map between Riemannian manifolds, weakly harmonic on $M \setminus A$. Then f is weakly harmonic on M .*

The result is established in §3, based on methods of Harvey-Polking [HP1]. We also present several situations in which we can apply regularity theory to assert that f is harmonic. In §4 we show how to construct certain harmonic maps with singularities in prescribed nonpolar sets.

Section 5 is devoted to a version of the Schwarz reflection principle for harmonic maps, generalizing that given by J. C. Wood in his thesis [W]; our proof is an adaptation of that in [HP1].

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2. Polar sets, capacities, and Hausdorff measures. We shall say that a set $A \subset M$ is *q-polar* ($1 < q < \infty$) if 0 is the only distribution in $L_{-1}^p(M)$ with compact support in A . (That notion coincides with $(1, q)$ -polar in the terminology of [L], where the concept was first brought to bear on problems of removable singularities.) q -polar is a concept independent of the Riemannian structure used to define it. Clearly A is q -polar iff $A \cap U_\alpha$ is q -polar for every set U_α of an open cover of M . Hence q -polar is a local concept; and it is appropriate to characterize it in Euclidean spaces. Furthermore, a set A is q -polar iff every compact subset is q -polar.

The *q-capacity* ($1 < q < \infty$) of a compact subset $K \subset \mathbf{R}^m$ is defined by

$$C_q(K) = \inf\{\|\phi\|_{q,1}^q\},$$

taking the infimum over all $\phi \in C_0^\infty(\mathbf{R}^m)$ for which $\phi \geq 1$ on K ; here $\|\cdot\|_{q,1}$ denotes