

Bounded Entire Solutions of Second Order Semilinear Elliptic Equations with Application to a Parabolic Initial Value Problem

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1. Introduction. In this paper we are concerned with the existence of positive entire solutions of second order semilinear elliptic equations of the type

$$(1) \quad \Delta u + f(x, u) = 0, \quad x \in \mathbb{R}^n,$$

where Δ is the n -dimensional Laplace operator, $n \geq 3$, and $f(x, u)$ is a continuous function on $\mathbb{R}^n \times (0, \infty)$. By an entire solution of equation (1) we mean a function $u \in C^2(\mathbb{R}^n)$ which satisfies (1) at every point of \mathbb{R}^n .

The importance of this problem in mathematical analysis, in geometry and in various applications has been recognized for many years; see, for example, the papers [1–3, 7, 8, 10–13, 16]. Most of the literature on the subject, however, has been devoted to “autonomous” equations of the form

$$(2) \quad \Delta u + f(u) = 0,$$

and it seems to us that very little is known about entire solutions of equations with genuinely “nonautonomous” structure. To the best of the authors’ knowledge, a significant contribution in the latter direction was first made by Ni [11, 12] who has established the existence of entire solutions for the following special cases of (1):

$$\Delta u + c(x)u^\gamma = 0, \quad \Delta u + c(x)e^u = 0.$$

The reader is referred to a paper by Kawano [7] in which considerable improvement and extension of Ni’s results have been obtained.

The objective of this paper is twofold. First, we extend Kawano’s existence theory so as to cover a much wider class of equations of the form (1). For this purpose the supersolution-subsolution method is employed. Secondly, we apply the results for (1) to establish the existence of positive global solutions of the Cauchy problem for semilinear parabolic equations of the form

$$(3) \quad u_t = \Delta u + f(x, t, u)$$