

# *Resonances in Scattering by a Resonator*

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**1. Introduction.** A number of mathematical descriptions of resonances for Schrödinger operators has been studied by different authors [5], [14], [15]. On the other hand, in the context of the Lax-Phillips Scattering Theory [11], the concept of scattering frequencies is considered; these complex frequencies present resonant properties for the wave motion in exterior regions.

Here we also deal with exterior problems and study the resonance phenomenon for the operator  $H = -\Delta_\Omega$ , the usual self-adjoint extension of the negative Laplace operator in an exterior domain  $\Omega \subset \mathbf{R}^3$  with Dirichlet boundary conditions. We shall assume that  $\Omega$  is a connected region with smooth boundary such that the obstacle  $0 = \mathbf{R}^3 - \Omega$  is compact.

With an adequate choice of units, the operator  $H = -\Delta_\Omega$  can be viewed as the Hamiltonian (energy observable) for the physical system which consists of a single quantum mechanical particle moving in  $\Omega$ . The states of this system can be represented by the unit vectors in the Hilbert space  $\mathcal{H} = L^2(\Omega)$ .

We denote by  $H_0$  the free Hamiltonian  $-\Delta$ , viewed as a self-adjoint operator on  $\mathcal{H}_0 = L^2(\mathbf{R}^3)$ .

It is physically clear and mathematically known (see for example [6], [12]), that there is a good scattering theory for the pair  $H_0, H$ . Therefore, the operator  $H$  is absolutely continuous and its spectrum consists of the interval  $[0, \infty)$ . In physical terms, this means that any state with Hamiltonian  $H$  is a scattering state and, as such, it must have a finite lifetime.

Following Lavine [8], [10], we characterize a resonance as a state  $\phi \in \mathcal{H}$  which is concentrated in a bounded region  $B \subset \Omega$  and which is almost an eigenvector of  $H$  in the sense that the spectral measure  $\langle \phi, E_H(\lambda) \phi \rangle$  of  $H$  at  $\phi$  is concentrated near a point  $\lambda_0 \in \sigma(H)$  (the corresponding resonant energy). By the results we establish in Section 2, it then follows that  $\phi$  will have unusually long sojourn and transit times.

In Section 3 we consider the case where the obstacle has a partially opened cavity and we provide explicit estimates for the spectral concentration of the resonances in terms of the width of the channel that connects the cavity with the exterior.

**2. Resonant states.** As in [8], we define the *energy width of a unit vector*  $\phi \in \mathcal{H}$  about a real number  $\lambda_0$  to be