

# *Scattering and Inverse Scattering for Steplike Potentials in the Schrödinger Equation*

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## **Introduction and Summary**

This paper studies both the forward and inverse problems in the scattering theory of the Schrödinger equation

$$(0.1) \quad -y'' + v(x)y = k^2y, \quad x \in \mathbf{R}$$

with “steplike” potentials  $v(x)$  which are asymptotic to different constants as  $x \rightarrow +\infty, -\infty$ . The forward problem is to define appropriate scattering data for potentials in a reasonable class of potentials and to study their properties; the inverse problem is twofold: to give sufficient conditions on candidate scattering data to assure that these data are indeed the scattering data of some potential and to give a method for constructing that potential from that scattering data.

Analogous problems on the half-line were studied by Agranovich and Marchenko [1]. This problem for potentials decaying to 0 at both  $+\infty$  and  $-\infty$  has been studied extensively by Faddeev [7] and by Deift and Trubowitz [6]. There are important applications of these constructions to the solution and analysis of the Korteweg-deVries equation [13], [3], [8].

The problem for steplike potentials on the full line was first studied by Buslaev and Fomin [2]. Following the methods of Faddeev [7], Buslaev and Fomin developed a set of conditions which they claimed were necessary and sufficient that candidate scattering data arise from a potential  $v$  such that

$$(0.2) \quad \int_{-\infty}^0 |v(x)|(1 + |x|)dx < \infty; \quad \int_0^{+\infty} |v(x) - c^2|(1 + |x|)dx < \infty.$$

A brief summary of the Buslaev-Fomin paper appears in the English translation [11] of a paper by Hruslov, who applies it to the Korteweg-deVries equation.

Unfortunately the paper of Buslaev and Fomin is incorrect: there is a counterexample [4] to their theorem, which is analogous to the counterexample to Faddeev’s theorem in [7], found by Deift and Trubowitz [6]. Deift and Trubowitz formulated a revised version of Faddeev’s theorem which could be proved cor-