

Energy Estimates for the Tensor Wave Equation in a Curved Spacetime

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1. Introduction. Let \square be the contraction of the second covariant derivative on a globally hyperbolic spacetime M . We shall obtain energy estimates for the Cauchy problem for the wave equation on a tensor field on M :

$$(1.1) \quad \square\omega = \rho.$$

These estimates yield an existence theorem in spaces of solutions of (1.1) for finite time. For the proof we shall use the technique in Choquet-Bruhat et al. [1]. In the same way we can obtain the energy estimates for the Cauchy problem for the wave equation on a differential form on M :

$$(1.2) \quad \Delta\omega = \rho$$

where Δ is the deRham Laplacian on M . Ellis-Hawking [3] gave energy estimates for the Cauchy problem for the tensor second order hyperbolic equation using a slightly different proof in a different setting, and their result is weaker than ours. Choquet-Bruhat et al. [1] gave energy estimates for the scalar wave equation (but not for the Cauchy problem) in a different setting. Our proof is simpler than that for the scalar case.

In some special spacetimes we shall also investigate $L^p - L^q$ estimates for the solution of the Cauchy problem (1.2) with a suitable modification. To the proof of the estimates we shall apply the technique in Miyachi [5].

I would like to thank Professor Robert S. Strichartz for useful advice and help. I am also grateful to Dr. A. Miyachi and the referee for valuable advice.

2. Energy inequalities. Let M be an $(n + 1)$ -dimensional spacetime with a Lorentzian metric g as follows: The manifold M is diffeomorphic to a product $R \times S$ where R is the real line and S is an n -dimensional submanifold of M and there is a C^∞ time function τ on M such that

$$\sum_{\mu} (\partial_{\mu}\tau)(\partial^{\mu}\tau) = -b < 0$$

everywhere. The function τ generates a foliation $\{M_t = \tau^{-1}(t) : t \in R\}$ and each