

Triangular Integration with Respect to a Nest Algebra Module

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1. Introduction. The concept of the triangular integral of an operator on Hilbert space with respect to a complete nest was introduced by Brodskii [3] and in short time it was developed considerably [4], [5], [7]. Erdos and Longstaff in [5] studied the triangular integral with respect to a fixed nest and gave many applications of it. Erdos in [4] continues the study of the triangular integral by giving new proofs of some fundamental convergence theorems in this theory. In this paper we introduce the notion of the triangular integral with respect to a nest algebra-module and characterize the set of operators for which this integral is convergent. We also study the convergence of this integral on symmetrically-normed ideals and prove that, when the given nest \mathcal{C} is continuous, every operator in the symmetrically-normed ideal \mathcal{C}_ω is the sum of two operators; one in the considered nest algebra-module $\mathcal{A}(\mathcal{C}, \sim)$ and one in the complementary module $\mathcal{A}(\mathcal{C}^\perp, \sim)$. This paper owes much to the papers of Erdos and Longstaff [5] and Erdos [4].

2. Notation and preliminaries. Standard terminology and notation will be used (see, for example [5], [6]). The terms *Hilbert space*, *subspace* and *operator* will be used to mean *complex Hilbert space*, *closed subspace* and *bounded linear operator* on a Hilbert space respectively. The set of all operators on a Hilbert space H will be denoted by $\mathcal{B}(H)$. A set \mathcal{C} of orthogonal projections is called a *nest of projections* if it is totally ordered by the usual ordering of operators. If a nest \mathcal{C} contains 0 and I and is complete as a lattice then \mathcal{C} is said to be a *complete nest*. For any member E of a complete nest \mathcal{C} , define

$$E^- = \bigvee \{F \in \mathcal{C} : F < E\}, \quad \text{when } E \neq 0, \text{ and}$$

$$E^+ = \bigwedge \{F \in \mathcal{C} : F > E\}, \quad \text{when } E \neq I.$$

By convention $0^- = 0$ and $I^+ = I$. If $E = E^-$ then \mathcal{C} is called a *continuous nest*. Also if $E \rightarrow \tilde{E}$ is an order homomorphism of \mathcal{C} into itself (that is $E \leq F$ implies $\tilde{E} \leq \tilde{F}$), define

$$E_* = \bigwedge \{\tilde{F} : F > E\} \quad (\text{for the notation cf. [6]}).$$

The set of all operators which leave invariant the range of each member of a nest