

Striated Solutions of Semilinear, Two-Speed Wave Equations

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Introduction. In [12], the authors showed that if P is a second order, strictly hyperbolic, scalar operator in one space dimension, then the propagation for singularities for the semilinear equation

$$(1) \quad Pu = f(u, Du)$$

is the same as the case when f is linear. That is, singularities do not interact to produce anomalous singularities and the regularity of u at any point $p_0 = (x_0, t_0)$ is determined completely by the regularity of the initial data at the two points where the backward characteristic curves from p_0 intersect the initial line. The purpose of this paper is to prove that this “linear” behavior persists in higher dimensions for a special class of operators and special initial data.

In §1 we introduce a class of hyperbolic operators, *two-speed* operators. For scalar operators we require the order to be less than or equal to two. More generally, we require that through any initial hypersurface Σ there pass exactly two characteristic surfaces for P . An additional hypothesis, valid for second order equations and symmetric hyperbolic systems is also imposed. The class of two-speed operators includes, in addition to the wave operator, many Lorentz invariant systems which arise in mathematical physics: Maxwell’s equations in a vacuum (wave operator form), the Pauli electron equation, the Dirac equation, the coupled Maxwell-Dirac and Klein-Gordon-Dirac equations.

Suppose that the initial data is infinitely differentiable parallel to a foliation Ψ near $0 \in \mathbf{R}^n$. We call such initial data *striated*. Striated initial data arise naturally in a variety of physical situations where the initial data is smooth in all but one variable. In addition to the obvious case of cartesian coordinates we mention data which is piecewise smooth and smooth in the angular variables in spherical coordinates (see [3], [16]) or data which is smooth except on a C^∞ , $(n - 1)$ -dimensional hypersurface (progressing waves). Notice that the singular support of striated data may be all of the hyperplane $t = 0$ so in that sense these data may represent quite complicated behavior.

Locally, \mathbf{R}^{n+1} is foliated by each of the two families of characteristic surfaces through the leaves of Ψ ; call these foliations \mathcal{F}_1 and \mathcal{F}_2 . The intersections of the characteristic surfaces in \mathcal{F}_1 and \mathcal{F}_2 give a local foliation, Φ , of \mathbf{R}^{n+1} near $(0,0)$.