

Characterizations of Classes of Functions Generated by Blocks and Associated Hardy Spaces

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Introduction

Let X denote Euclidean space \mathbf{R}^n or the fundamental cube, T^n , of \mathbf{R}^n , (viewed as the n -fold product of $(-1/2, 1/2]$) and let $|\cdot|$ denote Lebesgue measure on X . For $1 < q \leq \infty$ we say that a measurable function $b(x)$ is a q -block if there exists a cube $Q \subset X$ containing the support of b and such that

$$\left\{ \int |b(x)|^q dx \right\}^{1/q} \leq \frac{1}{|Q|^{1-1/q}}.$$

Classes of functions generated by q -blocks and numerical sequences $c = \{c_k\}$ satisfying

$$(1.1) \quad M(c) = \sum_{k \geq 1} |c_k| \left(1 + \log^+ \frac{1}{|c_k|} \right) < \infty$$

were introduced in [19] by M. Taibleson and G. Weiss in connection with a.e. convergence of Fourier series.

These classes, the *block spaces*, are “near” $L^1(X)$. They differ from the usual Lorentz and Orlicz classes (like $L^p(X)$ or $L \log^+ L(X)$) that are close to $L^1(X)$ in many ways. For example, there are no intermediate Orlicz classes between them and $L^1(X)$; moreover, if I is a finite interval in \mathbf{R} , any monotone function in $L \log^+ \log^+ L(I)$ belongs to B_∞ , the smallest of the block spaces. Here and in the following paper by Y. Meyer, M. Taibleson and Guido Weiss we will see many other examples of this feature, as well as certain differences among the block spaces: for example, in the latter paper it is shown that B_∞ is the only block space whose dual cannot be identified with L^∞ .

We define the space $B_q = B_q(X)$ as the class of all functions f which can be written as

$$(1.2) \quad f(x) = \sum_k c_k b_k(x),$$