

Continuous Glimm Functionals and Uniqueness of Solutions of the Riemann Problem

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1. Introduction

Let

$$(1.1) \quad u_t + f(u)_x = 0, \quad x \in \mathbf{R}, t > 0$$

with initial condition

$$(1.2) \quad u(x, 0) = u_0(x)$$

be a strictly hyperbolic system of conservation laws. We denote by D the differentiation operator; f is a three times continuously differentiable mapping from an open set \mathcal{U} of \mathbf{R}^N to \mathbf{R}^N ; the assumption of strict hyperbolicity means that $Df(u)$ has N distinct real eigenvalues for each u in \mathcal{U} :

$$(1.3) \quad \lambda_1(u) < \lambda_2(u) < \dots < \lambda_N(u).$$

We denote the right eigenvectors of $Df(u)$ by $r_i(u)$

$$(1.4) \quad Df(u)r_i(u) = \lambda_i(u)r_i(u),$$

and by $\ell_i(u)$ the left eigenvectors or eigenlinear forms of $Df(u)$

$$(1.5) \quad \ell_i(u)Df(u) = \lambda_i(u)\ell_i(u).$$

Those names are standard in the literature, but as a number of differentiations on compositions of operators and vectors depending on u will be performed, it is safe to have in mind that the $r_i(u)$ belong to the (trivial) tangent bundle to \mathcal{U} , and that the $\ell_i(u)$ belong to the (trivial) cotangent bundle of \mathcal{U} . These spaces cannot be identified, due to the lack of a canonical pairing, via an orthogonal structure.

The i -th characteristic field is called genuinely nonlinear if

$$(1.6) \quad D\lambda_i(u)r_i(u) \neq 0 \quad \forall u,$$

and linearly degenerate if

$$(1.7) \quad D\lambda_i(u)r_i(u) = 0, \quad \forall u.$$