

The Multicyclic n -tuples of an n -multicyclic Operator, and Analytic Structures on its Spectrum

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1. Introduction. Let $\mathcal{L}(\mathcal{X})$ denote the algebra of all (bounded linear) operators acting on the complex separable Banach space \mathcal{X} , and let \mathcal{A} be a subalgebra of $\mathcal{L}(\mathcal{X})$ containing the identity operator. The *multiplicity* of \mathcal{A} is the cardinal number

$$\mu(\mathcal{A}) = \inf \{ \text{card}(\Gamma) : \mathcal{X} = \bigvee [Ax : A \in \mathcal{A}, x \in \Gamma] \},$$

where \bigvee denotes “the closed linear span of.”

An operator $T \in \mathcal{L}(\mathcal{X})$ is called *n -multicyclic* if $\mu[\mathcal{A}(T)] = n$, where $\mathcal{A}(T)$ denotes the weakly closed algebra generated by T . Equivalently, there exists an n -tuple (y_1, y_2, \dots, y_n) in $\mathcal{X}^{(n)}$ (= the direct sum of n copies of \mathcal{X}) such that

$$\mathcal{X} = \bigvee \left\{ \sum_{j=1}^n p_j(T)y_j : p_j \text{ runs over all polynomials, } j = 1, 2, \dots, n \right\},$$

but \mathcal{X} is strictly larger than

$$\bigvee \left\{ \sum_{j=1}^{n-1} p_j(T)z_j : p_j \text{ runs over all polynomials, } j = 1, 2, \dots, n-1 \right\}$$

for all possible $(n-1)$ -tuples $(z_1, z_2, \dots, z_{n-1})$.

An n -tuple (y_1, y_2, \dots, y_n) with the above property is called a *multicyclic n -tuple* (for T).

Let $\mathcal{C}_n(T)$ denote the set of all multicyclic n -tuples of T , and let

$$\sigma_p^{(n)}(T^*) = \{ \lambda \in \mathbb{C} : \dim \ker(\lambda - T)^* = n \},$$

where T^* denotes the (Banach space) adjoint of the operator T . The topological properties of $\mathcal{C}_n(T)^-$ (= the closure of $\mathcal{C}_n(T)$ in $\mathcal{X}^{(n)}$) and those of $\sigma_p^{(n)}(T^*)$ are closely related: