

Analytic Operators and Spectral Decompositions

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1. Introduction. Let K be a compact subset of \mathbf{C}^n and let us consider the Fréchet space $A'(K)$ of all analytic functionals carried by K (see, for instance, [9], Chapt. IX). It is not necessarily true that $u \in A'(K_1) \cap A'(K_2)$ implies $u \in A'(K_1 \cap K_2)$. However, if one takes only compact subsets of \mathbf{R}^n (canonically embedded in \mathbf{C}^n), then such an implication is always true. Moreover, if K_1, \dots, K_m are arbitrary compact subsets of \mathbf{R}^n and $K = K_1 \cup \dots \cup K_m$, then the following decomposition holds:

$$(1.1) \quad A'(K) = A'(K_1) + \dots + A'(K_m).$$

This important result, which is a substitute for the existence of partitions of unity in the theory of hyperfunctions, has been proved by Martineau [11] (see also [16], [9]).

Equation (1.1) suggests the following question: Is the decomposition (1.1) a spectral decomposition? In other words, if $K \subset \mathbf{R}^n$ is a fixed compact set, is there any decomposable system of commuting linear operators on $A'(K)$ whose spectral decompositions are described by (1.1)? (We use the terminology from [20], but full details will be given in the next sections.) To give an answer to this question, an obvious candidate is the system of operators associated with multiplication by the coordinate functions (see Theorem 5.3).

Let us note from the very beginning that the general theory of spectral decompositions of commuting systems of linear operators on Fréchet spaces, as developed for instance in [20], cannot be directly applied. Indeed, if K_1, K_2 are compact sets, $K_1 \subset K_2$, then $A'(K_1) \subset A'(K_2)$ and the inclusion is continuous. Nevertheless, the space $A'(K_1)$ is not, in general, a closed subspace of $A'(K_2)$. Since the theory of spectral decompositions usually requires that the “involved” subspaces be closed, the above remark shows that it is necessary to make some adjustments to the general theory if one wants to include this important case.

The aim of this paper is to analyze the decomposition theorem for analytic functionals, more generally for analytic operators (see Definition 2.1), from the point of view of the theory of spectral decompositions of commuting systems of linear operators. To this end, we need a few abstract results concerning certain commuting systems of linear operators in Fréchet spaces, which might be of in-