

A Direct Construction for the Duals of Brown-Gitler Spectra

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Brown-Gitler spectra have had many applications in homotopy theory; however, the original construction, given in [3], was remarkably delicate. One of the purposes of this paper is to give a simpler argument that produces spaces that are Spanier-Whitehead duals of the original Brown-Gitler spectra. The reader should compare the results of J. Lannes given in [9].

Fix a prime p and let A^* be the mod p Steenrod algebra. Set U to be the category of graded unstable right A^* modules. If X is a CW complex of finite type, then $\tilde{H}_*(X, F_p) = \tilde{H}_*X$ is an object of U . If $M \in U$ and we wish to make the action map

$$M \otimes A^* \rightarrow M$$

a graded map, we must grade A^* negatively; that is, $\deg P^n = -2n(p - 1)$ and $\deg \beta = -1$ if $p > 2$, and $\deg Sq^n = -n$ if $p = 2$. *This convention will hold throughout the paper.* A^* acts on itself, on the right, by the usual multiplication. If M is a graded right A^* module, let ΣM be the graded module so that

$$\Sigma M_n = M_{n-1}.$$

Following Haynes Miller [10], define a functor

$$(\)_n : U \rightarrow F_p \text{ vector spaces}$$

by $(M)_n = M_n =$ elements of degree n . This functor is representable: there is an isomorphism

$$(1) \quad \Gamma : \text{Hom}_U(M(n), M) \rightarrow M_n$$

where

$$M(n) = \Sigma^n A^* / \{\beta^\epsilon P^i : 2(pi + \epsilon) > n\} A^* \quad \text{if } p > 2$$

or

$$M(n) = \Sigma^n A^* / \{Sq^i : 2i > n\} A^* \quad \text{if } p = 2.$$

Let $i_n \in M(n)_n$ be the top class. Then $\Gamma(f) = f(i_n)$. Because $(\)_n$ is an exact functor, $M(n)$ is projective in U . The modules $M(n)$ were studied, at $p = 2$, by