

Strong Differentiation of N -dimensional Subadditive Processes

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Introduction. The purpose of this paper is to prove a strong differentiation theorem which describes the local norm-convergence for positive N -dimensional subadditive processes in a UMB-lattice. The result does make it possible to give a unified treatment of the local mean behavior of operator averages for strongly continuous semigroups $\{S_t; t \geq 0\}$ of positive linear contractions on L_p ($1 \leq p < \infty$) and the differentiability of indefinite integrals of (B) -integrable UMB-lattice valued functions. However, the convergence of this kind for such processes has not received a general treatment as compared with the local pointwise convergence sometimes called the differentiation ([1], [2], [3], [4]). The fact is that much less is known of subadditive (or superadditive) local mean ergodic theorems even in L_p -spaces (cf. [9]). The norm-convergence of $t^{-1}(S_t - I)f$ ($t > 0$) as $t \rightarrow 0$ is usually studied in the analytical theory of semigroups in connection with the infinitesimal generators ([7], [8], [10]). Our result has the additional advantage that the norm-convergence of $t^{-1}(S_t - I)f$ as $t \rightarrow +0$ can also be deduced from our result as a special case under the boundedness condition $\sup\{t^{-1}\|(S_t - I)f\|: t > 0\} < \infty$. These applications will be mentioned at the end of this paper. The method of proof we have utilized in this work depends on the UMB-condition and the N -dimensional modification of a theorem of Hille and Phillips ([8], Theorem 7.11.1), so that the idea used by Derriennic and Krengel to prove the subadditive mean ergodic theorem in [6] has influence on the arguments.

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1. Preliminaries. Let \mathbf{R}_N ($N \geq 1$) be the Euclidean N -space equipped with the N -dimensional Lebesgue measure λ_N . For $u \in \mathbf{R}_N$, $\xi_i(u)$ denotes the i -th component of u for each i ($1 \leq i \leq N$). \mathbf{R}_N^+ is the positive cone of \mathbf{R}_N ,

$$\{u \in \mathbf{R}_N: \xi_i(u) \geq 0, 1 \leq i \leq N\},$$

and \mathbf{P}_N is the interior of the cone \mathbf{R}_N^+ , $\{u \in \mathbf{R}_N^+: \xi_i(u) > 0, 1 \leq i \leq N\}$. Given $u, v \in \mathbf{R}_N$, we write $u \leq v$ or $u < v$ according as $\xi_i(u) \leq \xi_i(v)$ or $\xi_i(u) < \xi_i(v)$ for each i ($1 \leq i \leq N$). If $u, v \in \mathbf{R}_N^+$ and $u < v$, then a left-half-open interval $I_{u,v} = (u, v]$ shall mean the set $\{x \in \mathbf{R}_N^+: u < x \leq v\}$. By \mathcal{J}_N^+ we denote the class of all