

Invariant Subspaces, Dilation Theory, and the Structure of the Predual of a Dual Algebra. II

C. APOSTOL, H. BERCOVICI, C. FOIAS
& C. PEARCY

1. Introduction. In the past few years operator theorists have been studying the structure of the preduals of various operator algebras, and thereby have made some remarkable advances in the areas of invariant subspaces, dilation theory, and reflexivity. (See the bibliography for a list of pertinent articles.) In this paper, which is a continuation of the sequence [12], [10], [6], we add to our knowledge of the structure of various preduals, and make some progress on the invariant subspace problem for contractions whose spectrum contains the unit circle. Most of the results herein were presented (by the fourth author) at the CBMS/NSF regional conference in Tempe, Arizona in May 1984.

The notation and terminology below agree with those in [12], [10], and [6]. For the benefit of the reader, we review a few pertinent definitions and earlier results. Let \mathcal{H} be a separable, infinite dimensional, complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators on \mathcal{H} . It is well known (cf. [21, p. 40]) that $\mathcal{L}(\mathcal{H})$ is the dual space of the ideal $\mathcal{C}_1(\mathcal{H})$ of trace-class operators under the bilinear functional

$$(1) \quad \langle T, L \rangle = \text{tr}(TL), \quad T \in \mathcal{L}(\mathcal{H}), L \in \mathcal{C}_1(\mathcal{H}).$$

A subalgebra \mathcal{A} of $\mathcal{L}(\mathcal{H})$ that contains $1_{\mathcal{H}}$ and is closed in the weak* topology on $\mathcal{L}(\mathcal{H})$ is called a *dual algebra*. It follows from general principles that if \mathcal{A} is a dual algebra, then \mathcal{A} is the dual space of the quotient space $Q_{\mathcal{A}} = \mathcal{C}_1(\mathcal{H}) / {}^{\perp}\mathcal{A}$ (where ${}^{\perp}\mathcal{A}$ is the preannihilator in $\mathcal{C}_1(\mathcal{H})$ of \mathcal{A}) under the pairing

$$(2) \quad \langle T, [L] \rangle = \text{tr}(TL), \quad T \in \mathcal{A}, [L] \in Q_{\mathcal{A}}.$$

(Throughout the paper we write $[L]_{Q_{\mathcal{A}}}$, or simply $[L]$ when there is no possibility of confusion, for the image of a trace-class operator L in the quotient space $Q_{\mathcal{A}}$.) If x and y are vectors in \mathcal{H} , then the associated rank-one operator $x \otimes y$ is defined by $(x \otimes y)(u) = (u, y)x$, $u \in \mathcal{H}$. Clearly $x \otimes y \in \mathcal{C}_1(\mathcal{H})$. Since every operator L in $\mathcal{C}_1(\mathcal{H})$ can be written as $L = \sum_{i=1}^{\infty} x_i \otimes y_i$ for certain square summable se-