# *An Elementary, Direct Proof of Partial Regularity for Solutions of Certain Nonlinear Elliptic Systems*

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**1. Introduction.** Partial regularity assertions for weak solutions of nonlinear elliptic systems have mostly been proved using one of two by now standard methods. First historically was the "indirect" or "blow up" technique developed by DeGiorgi, Federer and Almgren for geometric measure theory and adapted to PDE by Giusti-Miranda [6]. The idea here consists in proving estimates via a contradiction argument. Indeed, were some desired inequality false, there would exist a sequence of solutions which fail to satisfy the estimate and which, appropriately rescaled, converge to the solution of some nice linearized problem. As this limit function does satisfy the correct inequality, we hope to get a contradiction to the assumed failure of the estimate for each of the convergent functions. This technique thus relies on various compactness arguments and does not in general yield explicitly computable constants. See Chapter IV in Giaquinta [2] for a full elaboration of these ideas.

The second basic scheme for proving partial regularity is the so-called "direct method," which attempts instead to fashion explicit estimates. An early paper justifying this approach was Giaquinta-Giusti [3], which derives such bounds from "reverse Hölder inequality" estimates. Subsequent work of Giaquinta, Giusti, Modica, Campanato, and many others has greatly extended the applicability of this technique: see Chapters V–VIII of Giaquinta [2] for details and references.

Our main contribution in this paper is to show that the direct method can be applied to certain quasilinear systems by elementary arguments, without our invoking the reverse Hölder inequalities developed in Gehring [1], Giaquinta-Modica [4], Stredulinsky [7], etc. This seems to us worthwhile for several reasons. Primarily, the theory brought forth in [1], [4], [7], etc. is rather subtle and technical, especially the estimates for bounded domains. It therefore should be useful to discover alternative techniques in the hopes that these may be easier to apply in new circumstances, for example to parabolic PDE, equations with constraints, degenerate PDE, etc. All this being said, we of course recognize that the method of Gehring et al. gives better estimates and is widely applicable to other problems in analysis and PDE.



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