

A Weighted Inequality and Eigenvalue Estimates for Schrödinger Operators

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1. Introduction. For $v(x) \geq 0$ on R^n , $n \geq 3$, C. L. Fefferman and D. H. Phong [5] have estimated the eigenvalues of the Schrödinger operator $H = -\Delta - v$ in terms of the functional

$$Y_p(Q, v) = |Q|^{2/n} \left(\frac{1}{|Q|} \int_Q v^p \right)^{1/p},$$

defined for $p \geq 1$ and Q a cube in R^n . For $\lambda \geq 0$, they show that a lower bound on the number of eigenvalues $\leq -\lambda$ is given by the largest number of cubes Q_1, \dots, Q_N that can be found with pairwise disjoint doubles, side length at most $\lambda^{-1/2}$ and $Y_1(Q_j, v) \geq C$, $1 \leq j \leq N$. If $p > 1$, a corresponding upper bound is given by (a multiple of) the number of minimal dyadic cubes Q of side length at most $\lambda^{-1/2}$ satisfying $Y_p(Q_j, v) \geq C_p$. Later, in [7] and [8], R. Kerman and the author obtained analogous estimates with the functionals Y_p replaced by the functional

$$Z(Q, v) = \frac{1}{|Q|_v} \int_Q I_2(\chi_Q v) v,$$

where

$$I_2 f(x) = \int |x - y|^{2-n} f(y) dy$$

denotes the Newtonian potential of f . As shown in [8],

$$Y_1(Q, v) \leq CZ(Q, v) \leq C_p \sup_{Q' \subset Q} Y_p(Q', v) \quad \text{for } p > 1.$$

For other results along these lines see Chang, Wilson and Wolff [2], and Chanillo and Wheeden [3]. The starting point for obtaining these estimates is to find