Helical Submanifolds in Euclidean Spaces

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1. Introduction. Let M^n be a connected n-dimensional ($n > 1$) submanifold of an m-dimensional Euclidean space E^m. If every geodesic of M, considered as a curve in the ambient space, has all Frenet curvatures constant and these curvatures are independent of the chosen geodesic, then M^n is called a helical submanifold of E^m [8]. A helical submanifold is said to be of order d if each geodesic is properly contained in a d-plane of E^m. Helical submanifolds were investigated in [3], [4], [5], [7], [8], [9]. In [3], [4] Chen and Verheyen classified helical surfaces in E^m ($m = 3,4,5$). They showed that a helical surface in E^m ($m = 3,4,5$) is a 2-plane or the second standard immersion of a 2-sphere (i.e., a 2-sphere or a Veronese surface). In [5], Houh and the author proved that there is no surface M helically immersed into E^m of order 3.

Several years ago, B. Y. Chen conjectured that any helical submanifold of E^m is either spherical or an open portion of an n-plane. In this article, we show that any compact helical submanifold in E^m with a transitive isometry group is spherical. We also show that a helical surface of order 4 must be of positive constant curvature, and classify all helical immersions of compact rank one symmetric spaces.

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2. Notations and basic formulas. Assume that M is an n-dimensional submanifold immersed into E^m. Let V and $\tilde{V}$ be the covariant derivatives of M and E^m, respectively. For any two vector fields X, Y tangent to M, the second fundamental form h is given by

$$h(X,Y) = \tilde{\nabla}_X Y - \nabla_X Y.$$

For any vector field normal to M, we put

$$\tilde{\nabla}_X \xi = -A_\xi X + \nabla^\perp_X \xi,$$

where $-A_\xi X$ and $\nabla^\perp_X \xi$ denote the tangential and normal components of $\tilde{\nabla}_X \xi$ respectively. Then $\nabla^\perp$ is called the normal connection of the normal bundle $N(M)$.

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