

# *Second Fundamental Form for Varifolds and the Existence of Surfaces Minimising Curvature*

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## 1. Introduction

The Plateau problem is concerned with the existence and regularity of  $m$ -dimensional “surfaces”  $M$  in  $\mathbf{R}^p$  having prescribed boundary and least mass. Thus one requires  $M$  with prescribed boundary such that  $\int_M 1$  is a minimum. One obtains related problems by replacing  $\mathbf{R}^p$  by a Riemannian manifold  $N$ ; we can also impose other constraints on  $M$  such as orientability and membership in some homology class of  $N$ .

Instead of seeking to minimise  $\int_M 1$ , we may try to minimise  $\int_M F(|\mathcal{S}M|)$ , where  $\mathcal{S}M$  is the second fundamental form of  $M$  (suitably interpreted if  $M$  is not  $\mathbf{C}^2$ ) and  $F$  is positive and convex. In terms of the number of “differentiations” performed in the integrand, this is the next natural, geometrically invariant, problem after the Plateau problem. In this paper we will give a reasonably complete solution to the existence question for this class of problems. In the process, we develop some concepts of independent interest, including a notion of second fundamental form which applies to a general class of integral varifolds.

We now outline the main results and discuss our approach.

**Section 2.** We recall some standard terminology, see also [FH] and [AW].

**Section 3.** We recall the basic properties of varifolds and introduce the idea of an oriented varifold, and in particular of oriented integral and rectifiable varifolds. We prove a compactness theorem (3.1) for oriented integral varifolds analogous to the usual compactness theorems for integral varifolds and integral currents.

**Section 4.** We introduce the concept of a measure-function pair  $(\mu, f)$ , i.e. a Radon measure  $\mu$  together with a vector-valued function  $f \in \mathbf{L}_{\text{loc}}^1(\mu)$ . We discuss what it means for a sequence  $\{(\mu_k, f_k)\}_{k=1}^\infty$  of such pairs to converge in a weak or a strong sense—in case  $\mu_k = \mu$  for all  $k$  this is equivalent to the usual notions of weak or strong convergence of  $\{f_k\}$  relative to the measure  $\mu$ . By introducing a