

# Some Remarks on Singular Integrals

XIANLIANG SHI

## §1. Introduction

In this paper we will suppose throughout that  $x \in R^n$  ( $n \geq 1$ ) and  $K(x)$  is a function defined on  $R^n \setminus \{0\}$  which satisfies the following cancellation condition

$$(1) \quad \int_{R_1 < |x| < R_2} K(x) dx = 0 \quad (0 < R_1 < R_2 < \infty)$$

and the Hörmander condition

$$(2) \quad \int_{|x| > 2|y|} |K(x+y) - K(x)| dx \leq B \quad (|y| \neq 0).$$

Let  $b(x) = b_0(|x|)$  be a bounded radial function and let  $H(x) = K(x)b(x)$ . We will discuss boundedness of the convolution operator

$$Tf(x) = \text{p.v. } H * f(x)$$

in the  $L^p(R^n)$  sense. For the function  $K(x)$  we will also suppose that it either satisfies

$$(3) \quad K(x) = \frac{\Omega(x)}{|x|^n},$$

where  $\Omega(x)$  is a homogeneous function of degree zero, or

$$(3') \quad K(x) = O\left(\frac{1}{|x|^n}\right).$$

The aim of the present paper is to discuss the following problem.

**Problem.** Let  $K(x)$  satisfy conditions (1), (2) and (3) or (1), (2) and (3'). Is the operator  $T$  bounded in the space  $L^2(R^n)$ ?

This problem was proposed to me by Professor A. Torchinsky and is connected with the work [1] by R. Fefferman. For  $n \geq 2$  and operators satisfying conditions (1), (2) and (3) we will give the affirmative answer to the problem in §2. It is well known that the answer is negative in the case  $n = 1$  (see [2]).

In order to study the operator  $T$  satisfying conditions (1), (2) and (3') we in-