

Normal Families and Fix-points of Meromorphic Functions

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1. Introduction

Let $f(z)$ be a meromorphic function in a region D and z_0 a point of D . If $f(z_0) = z_0$, then z_0 is a fix-point of $f(z)$. There are many papers on fix-points of entire and meromorphic functions (cf. [1]–[5]). It seems to me, however, that the connection between the normality of a given family of holomorphic or meromorphic functions and the lack of fix-points of both these functions and their derivatives has not been studied.

The principal aim of this paper is to prove the following theorem.

Theorem 1. *Let \mathcal{F} be a family of meromorphic functions in a region D and k be a positive integer. If, for every function $f(z)$ of \mathcal{F} , both $f(z)$ and $f^{(k)}(z)$ (the derivative of order k) have no fix-points in D , then \mathcal{F} is normal there.*

For the proof of Theorem 1, Sections 2 and 3 are devoted to the case of $k = 1$, which is the most important. Then, in Section 4, we give a brief formulation of the case $k \geq 2$.

2. Preliminary Lemmas

Lemma 1. *Suppose that $f(z)$ is meromorphic in $|z| < R$ ($0 < R \leq \infty$). If $f(0) \neq 0, \infty$; $f'(0) \neq d$ and $df''(0) - f'(0) \neq 0$, then*

$$(2.1) \quad T(r, f) < \bar{N}(r, f) + N\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{f' - (z + d)}\right) \\ - N\left(r, \frac{1}{(z + d)f'' - f'}\right) + S(r, f)$$