

Weighted Spaces of Entire Functions

F. HASLINGER

In this paper weight functions of the form $\exp(-p_m(z))$ are considered, where $p_m: \mathbf{C}^n \rightarrow \mathbf{R}$ are convex functions, $m \in \mathbf{N}_0$, with certain growth conditions, guaranteeing that the corresponding Fréchet spaces of entire functions are nuclear and that the dual space can be identified with certain (DF) -spaces of entire functions by means of the Fourier-Laplace transform. Nonradial weight functions are of special interest because one cannot use the Taylor series expansions for entire functions in these cases. Finally the problem of existence of bases in these spaces is investigated.

0. Introduction. Let $P = (p_m)_{m=0}^\infty$ be a family of measurable functions $p_m: \mathbf{C}^n \rightarrow \mathbf{R}$ such that $p_m(z) \geq p_{m+1}(z)$ for each $z \in \mathbf{C}^n$ and for each $m \in \mathbf{N}_0$. Suppose further that the function $\exp \circ p_0$ is locally integrable. Let \mathcal{F}_P denote the space of all entire functions $f: \mathbf{C}^n \rightarrow \mathbf{C}$ such that

$$\|f\|_m := \sup\{|f(z)| \exp(-p_m(z)) : z \in \mathbf{C}\} < \infty$$

for each $m \in \mathbf{N}_0$. \mathcal{F}_P is endowed with the topology determined by the system of the norms $(\|\cdot\|_m)_{m=0}^\infty$. In this way \mathcal{F}_P becomes a Fréchet space the topology of which is in general stronger than the topology of uniform convergence on the compact subsets of \mathbf{C}^n (see also [5]). First it is shown that under certain growth conditions for the weight functions p_m the space \mathcal{F}_P is nuclear. Some special cases can be found in [1] and [23]. We only outline the proof of nuclearity here, because most of the steps are straightforward.

The next part of this paper is devoted to the representation of the dual space \mathcal{F}'_P of \mathcal{F}_P as a (DF) -space of entire functions. In this connection it is important to assume that the weight functions p_m are convex so that one can use the Fourier-Laplace transform of continuous linear functionals on \mathcal{F}_P in order to derive the representation of \mathcal{F}'_P as the (DF) -space of all entire functions F such that

$$\|F\|_m^* := \sup\{|F(w)| \exp(-p_m^*(w)) : w \in \mathbf{C}^n\} < \infty$$

for an $m \in \mathbf{N}_0$, where p_m^* is the conjugate function of p_m . Special cases of such representations (with radial symmetric weight functions) first appeared in a paper of L. Ehrenpreis [3], where the method is used to identify functions with the sequence of its Taylor coefficients which cannot be applied for spaces with more general, especially nonradial functions. Some other results in this direction can