

# On Certain Inductive Limit $C^*$ -Algebras

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It has been suggested by E. G. Effros [6] that a study of inductive limits of  $C^*$ -algebras of the form  $\mathcal{C}(X) \otimes F$  (with  $F$  finite-dimensional  $C^*$ -algebra), as a generalization of A.F.-algebras, may be of interest.

Indeed, such algebras are, for instance, those considered by J. Bunce and J. Deddens (in [3]).

The present paper contains general results concerning inductive limits of algebras of the form  $\mathcal{C}(X) \otimes F$  as well as a structure theorem for a certain class of inductive limits of  $C^*$ -algebra of the form  $\mathcal{C}(\mathbf{T}^2) \otimes M_n$ .

For each positive integer  $p \geq 1$ , let  $\mathcal{C}^*(S(p))$  be the  $C^*$ -algebra generated by all weighted shifts (with respect to some fixed orthonormal basis  $(e_m)_{m \geq 0}$  of the Hilbert space  $\mathcal{H}$ ) of period  $p$ .

Given a strictly increasing sequence of positive integers  $(n_k)$ , with  $n_k$  dividing  $n_{k+1}$  for all  $k \geq 1$ , the Bunce-Deddens algebra, denoted in this paper by  $\mathcal{A}((n_k))$ , is  $\bigcup_k \overline{\nu(\mathcal{C}^*(S(n_k)))}$  (see [3]), where  $\nu: B(\mathcal{H}) \rightarrow B(\mathcal{H})/K(\mathcal{H})$  is the canonical surjection onto the Calkin algebra.

It follows from ([2], proof of Theorem 2.2) that, for  $p \geq 2$ ,  $\nu(\mathcal{C}^*(S(p)))$  is  $*$ -isomorphic to  $M_p(\mathcal{C}(\mathbf{T})) \equiv$  the algebra of all  $p \times p$  matrices whose entries are continuous functions on the unit circle  $\mathbf{T}$ . Under this isomorphism, if  $S$  is the weighted shift given by  $Se_m = d_{m+1}e_{m+1}$  ( $m \geq 0$ ), where  $d_{m+p} = d_m$  for all  $m \geq 0$ , then  $\nu(S)_{1,p}(z) = d_p z$ ,  $\nu(S)_{i+1,i}(z) = d_i I$  for  $1 \leq i \leq p-1$  and  $\nu(S)_{i,j} = 0$  for all other  $i, j$ .

Using these  $*$ -isomorphisms, each  $\mathcal{A}((n_k))$  is  $*$ -isomorphic to the inductive limit of:

$$\mathcal{C}(\mathbf{T}) \otimes M_{n_1} \xrightarrow{\Psi_1} \mathcal{C}(\mathbf{T}) \otimes M_{n_2} \xrightarrow{\Psi_2} \dots$$

where for each  $k$ ,  $\Psi_k$  is a *particular* isometric  $*$ -homomorphism which is *compatible* with the covering map  $\mathbf{T} \ni z \mapsto z^{n_{k+1}/n_k} \in \mathbf{T}$  (in the sense of Definition 2.2 below).

We shall now state the main result of this paper.

We fix two strictly increasing sequences of positive integers  $(p_k)$ ,  $(q_k)$ , with  $p_k$  (resp.  $q_k$ ) dividing  $p_{k+1}$  (resp.  $q_{k+1}$ ) for all  $k \geq 1$ .

We consider systems of the form: