

Curvature and Focal Points in Riemannian Foliations

JAMES J. HEBDA

Introduction. Curvature strongly influences the topology of a complete Riemannian manifold. This can be observed in Myer's Theorem where a positive lower bound on Ricci curvature implies compactness of the universal cover, or in the Cartan-Hadamard Theorem where non-positive sectional curvature implies the universal cover is diffeomorphic to euclidean space. In these two theorems, curvature acts through the medium of conjugate points, the one or other assumption on curvature implying the existence or non-existence of conjugate points and thereby the conclusion concerning the universal cover. In its own right, the conjugate locus has a prominent place in Riemannian geometry. A manifold possessing conjugate loci similar to those of a compact rank one symmetric space shares topological and geometrical properties with the symmetric space. On this account there are theorems of Klingenberg [12], of Warner [19], and the Blaschke conjecture [1].

A foliation F of a manifold M is a Riemannian foliation if there exist a covering of M by open sets U_i and submersions $f_i: U_i \rightarrow N$ where N (called the model space) is a Riemannian manifold such that the leaves of $F|_{U_i}$ are the level sets of f_i and the diffeomorphisms

$$g_{ij}: f_j(U_i \cap U_j) \rightarrow f_i(U_i \cap U_j)$$

satisfying $f_i = g_{ij} \circ f_j$ are isometries. A Riemannian metric on M is bundle-like if the f_i are Riemannian submersions. Every Riemannian foliation admits bundle-like metrics. By the sectional or Ricci curvature of F we mean the sectional or Ricci curvature of the model space N .

A p -form ω on M is base-like for F if $\omega|_{U_i} = f_i^* \omega_i$ where ω_i is a p -form on $f_i(U_i)$. The collection of all base-like forms constitute a subcomplex of the de Rham complex of forms on M whose homology, denoted $H^*(M/F)$, is called the base-like cohomology of F [16].

By a theorem of O'Neill [15] on Riemannian submersions, the focal points of a leaf of a Riemannian foliation with bundle-like metric can be related to conjugate points in the model space. This idea underlies the results of this paper. The first is an analogue of Myers' Theorem.

Theorem 1. *Let M, F be a Riemannian foliation with complete bundle-like*