

Commutators in Ideals of Trace Class Operators II

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As its title indicates, this note is a continuation of the preceding paper [1]. The reader is referred to it for the notations and definitions that are used here. The main result in this work is as follows.

Theorem 1. *If $p < 1$, then*

$$[\mathcal{C}_p, \mathcal{B}(\mathcal{H})] = [\mathcal{C}_{2p}, \mathcal{C}_{2p}] = \mathcal{C}_p^0.$$

This theorem together with the results of [1] immediately yields the following.

Corollary 2. *If $p < 1$ and $J \in \mathcal{C}_p$, then*

$$1 + J \in \{1 + \mathcal{C}_p, \mathcal{B}(\mathcal{H})\}$$

if and only if $\det(1 + J) = 1$.

Proof. The assertion is implied by Theorem 1 and [1, Theorem 19].

Corollary 3. *If $p < 1$, then the map*

$$\det_* : K_1(\mathcal{C}_p, \mathcal{B}(\mathcal{H})) \rightarrow \mathbf{C}^*$$

is an isomorphism.

Proof. As observed in [1], \det_* is surjective. By Theorem 20 of [1] the kernel of \det_* is isomorphic to the additive group of

$$\frac{\mathcal{C}_p^0}{[\mathcal{C}_p, \mathcal{B}(\mathcal{H})]}$$

and by Theorem 1 this space is 0. Hence \det_* is also injective. \square

It is convenient to begin the proof of Theorem 1 with a lemma. If J is a self-adjoint compact operator, write $\|J\|_p^p$ for the sum of the p -th powers of the absolute values of its eigenvalues. Thus, $J \in \mathcal{C}_p$ if and only if $\|J\|_p^p < \infty$.